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Optimization of micro-heat sink based on theory of entropy generation in laminar forced convection

Pallavi Rastogi, Shripad P. Mahulikar*

Department of Aerospace Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

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ABSTRACT

The entropy generation ($\dot{S}_{gen}$) due to fluid friction and convective heat transfer is studied *en-route* the microscale. The $\dot{S}_{gen}$ for water-flow through a circular tube for the constant wall heat flux boundary condition is estimated. The number of tubes (N) en-route the microscale is increased by correspondingly decreasing each tube diameter, for fixed total mass flow rate and the total heat flow rate is also kept constant. There exists an optimum tube diameter ($D_{N,opt}$) and a corresponding optimum natural number N (N_{opt}) at which, the sum-total $\dot{S}_{gen}$ is minimum. Criterion for $D_{N,opt}$ is obtained in terms of Reynolds number and a modified Brinkman number, which shows that $D_{N,opt}$ depends only on the total heat flow rate. Unlike other reported studies, the fluid temperature in the denominator of the entropy generation terms is considered as local and variable. The difference in $\dot{S}_{gen}$ based on reported studies and this investigation increases significantly especially towards the microscale.

1. Introduction

The analyses based on the 2nd Law of Thermodynamics have infused in to Fluid Mechanics and Heat Transfer, due to the emphasis on energy quality conservation [1]. These analyses are the bases for estimating the irreversibility (entropy generation) associated with heat transfer processes. Entropy generation due to irreversibilities in processes is responsible for reducing the available work, by reducing the thermodynamic efficiency. Bejan [2] described 2nd Law analysis for the design of thermal systems and gave the procedure for minimizing the entropy generation rate ($\dot{S}_{gen}$). Herwig [3] discussed three aspects of analysis based on $\dot{S}_{gen}$ minimization in convective heat transfer process: (*i*) conceptual considerations, viz. the quality of energy lowered; (*ii*) determination of entropy generation from velocity and temperature fields; (*iii*) $\dot{S}_{gen}$ based assessment using the Darcy friction factor, $f_D = (\Delta P{\cdot}D)/(\rho_m{\cdot}u_m^2{\cdot}l)$, and the convective heat transfer coefficient, h. The Entropy Generation Minimization (EGM) is the minimization of the total $\dot{S}_{gen}$ ($\dot{S}_{gen,tot}$) in a finite space and finite time process with competing irreversibilities [4] and is the basis for optimal design of thermal systems [5,6]. In convective heat transfer process, the combined irreversibilities due to fluid friction and fluid conduction heat transfer determine $\dot{S}_{gen,tot}$.

The sum-total volumetric entropy generation rate due to fluid conduction heat transfer and fluid friction is given as,

$$\dot{S}'''_{gen,tot} = k\left(\frac{\nabla T}{T}\right)^2 + \mu\frac{\Phi}{T}. \tag{1}$$

In Eq. (1), Φ is the viscous dissipation function, k is thermal conductivity, μ is dynamic viscosity, and ∇T is the temperature gradient. For laminar fully developed forced convection in a circular tube with axisymmetry, Eq. (1) for the cylindrical coordinates reduces to,

$$\dot{S}'''_{gen,tot} = \frac{k}{T^2}\cdot\left[\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] + \frac{\mu}{T}\cdot\left(\frac{\partial u}{\partial r}\right)^2. \tag{1.1}$$

Thus, depends on the gradients of T and u, the absolute temperature of the fluid, T (in K), and the fluid properties (μ and k). In all the reported studies so far, the fluid temperature terms in the denominator of Eq. (1.1) are fixed at an invariant reference value, T_{ref}. This approximation is valid for small temperature variations, which is generally valid for the conventionally-sized tubes:

(i) $(\partial T/\partial r){\cdot}\Delta r \ll T_{ref}$.
(ii) $(\partial T/\partial z){\cdot}\Delta z \ll T_{ref}$. Axial variation of fluid temperature can become important towards the microscale (decreasing tube diameter, D), because the ratio [7],

$$[(\partial T/\partial z)/(\partial T/\partial r)] \propto (1/D). \tag{1.2}$$

* Corresponding author.
E-mail addresses: pallavirastogi@aero.iitb.ac.in (P. Rastogi), spm@aero.iitb.ac.in (S.P. Mahulikar).

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Nomenclature

A_{cs}	cross sectional flow area [m^2]
A_{sur}	surface area [m^2]
C_p	specific heat at constant pressure [J kg^{-1} K^{-1}]
D	tube diameter [μm]
D_h	hydraulic diameter [m]
f_D	Darcy friction factor [−]
h	heat transfer coefficient [W m^{-2} K^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
l	tube length [cm]
$\dot{m}$	mass flow rate [kg s^{-1}]
N	number of tubes [−]
P	static pressure [Pa]
q	convective heat transfer rate [W]
q_w''	wall heat flux [W cm^{-2}]
r	radial direction [m]
R	tube radius [μm]
$\dot{S}_{gen}$	global entropy generation rate [W K^{-1}]
	volumetric entropy generation rate [W m^{-3} K^{-1}]
T	fluid temperature [K]
u	axial velocity [m s^{-1}]
z	axial (flow) direction [m]

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
Φ	viscous dissipation function [W m^{-3}]
μ	dynamic viscosity [Pa-s]
ρ	density [kg m^{-3}]

Non-dimensional numbers

Be^*	Bejan number (in this study) based on global entropy generation rate ($\dot{S}_{gen,HT}/\dot{S}_{gen,tot}$)
$Br_{q_w''}$	Brinkman number based on q_w'' $((\mu\cdot u_m^2)/(q_w''\cdot D_h))$
Nu	Nusselt number ($h\cdot D/k$)
Pe	Peclet number ($u_m\cdot D_h/\alpha$)
Pr	Prandtl number ($\mu\cdot C_p/k$)
Π_ν	Modified Brinkman number based on $q_{w,tot}$ ($\mu\, u_m^2\cdot D_{N,opt}/q_{w,tot}$)
Re	Reynolds number ($\rho\cdot u_m\cdot D_h/\mu$)
St	Stanton number ($h/(\rho\cdot u_m\cdot C_p)$)

Subscripts

ax	axial (z) direction
ex	exit
fr	fluid friction
HT	heat transfer
in	inlet
m	bulk mean
max	maximum
min	minimum
N	N number of tubes
1	single tube
opt	optimum
rad	radial (r) direction
ref	reference
tot	total
w	wall
0	inlet

Abbreviations

BC	Boundary Condition
EGM	Entropy Generation Minimization

1.1. Entropy generation in internal laminar forced convective macro-flow

Several investigations will now be reported to study $\dot{S}_{gen}$ in steady-state forced convection with constant fluid properties through channels of different geometries. Bejan [8] analytically studied $\dot{S}_{gen}$ for fully developed forced convection in a circular tube, for the constant wall heat flux, q_w'' = const. boundary condition (BC). The axial conduction in the fluid was considered negligible relative to radial conduction, i.e. $(\partial T/\partial z) \ll (\partial T/\partial r)$. For given mass flow rate, $\dot{m} \propto u_m\cdot D^2$, Reynolds number, $Re_D \propto D^{-1}$; hence, the criterion for the optimum tube radius for minimum $\dot{S}_{gen}$ ($\dot{S}_{gen,min}$) was obtained for laminar flow as, $Re_{D,opt} = 0$. Thus, optimum D (D_{opt}) is large enough for $\dot{S}_{gen,min}$ to be determined mainly by the fluid conduction irreversibilities. Nag & Kumar [9] numerically analyzed the 2nd Law for incompressible forced convection of air in a circular tube with, q_w'' = const. BC. The dimensionless total entropy generation rate, ψ [$= \dot{S}_{gen}/(\dot{m}\cdot C_p)$] was derived in terms of: (*i*) the dimensionless temperature difference defined as, $\tau(z) = [T_w(z) - T_m(z)]/T_{m,0}$; (*ii*) the ratio of convective heat transfer to fluid pumping power (flow work) $= q/(A\cdot u_m\cdot\Delta P)$. The optimum values of $[\tau(z=0)-\tau(z=l)]$, $q/(A\cdot u_m\cdot\Delta P)$, and fluid mean velocity (u_m) were obtained, which correspond to $\dot{S}_{gen,min}$. Sahin [10] studied entropy generation of water flow for, q_w'' = const. BC; and obtained, ψ[Re(u_m),τ]. For constant $T_{m,0}$, as Re(u_m) is increased, the contribution of fluid friction in determining ψ was found to dominate over the contribution of convective heat transfer. Circular geometry, which has the least surface area (A_{sur}) for given cross sectional flow area (A_{cs}) has the lowest ψ (in which, contribution of fluid friction is dominant); hence, it is the best choice for high Re(u_m)-flows. Sekulic *et al.* [11] used mass, energy and entropy balances, to mathematically show that, $\psi = f[\tau_1, (l/D_h), Re, Br_{Tw}]$; where, Br is Brinkman number and the inlet temperature ratio, $\tau_1 = [1-(T_{m,0}/T_w)]$, for T_w = const. BC. Sahin [12] studied entropy generation in convective flow of water and glycerol in a circular duct for, T_w = const. BC. The ψ was plotted versus the modified Stanton number, $St' = St\cdot(l/D)$, and τ_1. Sahin [13] performed a similar study for q_w'' = const. BC and defined the inlet temperature ratio as, $\tau_2 = [(T_{w,0}/T_{m,0})-1]$. Both the studies by Sahin [12,13] showed that ψ increases with increasing St' and inlet temperature ratio. The role of increased inlet temperature ratio is to increase the temperature difference between the wall and the fluid, which increases the entropy generation due to convection. Further, it is inferred that the increase in St' increases the entropy generation due to convection, due to increase in the cross-sectional dimension. Mahmud & Fraser [14] studied $\dot{S}_{gen}$ in fully developed forced convection in a circular tube for, q_w'' = const. BC. They analytically derived the entropy generation and Bejan number Be, which was defined locally based on the volumetric entropy generation rate as,

$$Be = \frac{\dot{S}'''_{gen,HT}}{\dot{S}'''_{gen,tot}} = \frac{\dot{S}'''_{gen,HT}}{\dot{S}'''_{gen,HT} + \dot{S}'''_{gen,fr}} = \frac{1}{1+\left(\frac{\dot{S}'''_{gen,fr}}{\dot{S}'''_{gen,HT}}\right)}. \tag{2}$$

In Eq. (2), due to fluid friction is $\dot{S}'''_{gen,fr}$, due to fluid conduction heat transfer is $\dot{S}'''_{gen,HT}$ (it includes due to radial and axial heat conduction), and $\dot{S}'''_{gen,tot} = \dot{S}'''_{gen,HT} + \dot{S}'''_{gen,fr}$. The ratio in the bracket is a monotonic increasing or decreasing function of Brinkman number based on q_w'' $\left(Br_{q_w''}\right)$, depending on whether the viscous dissipation term is considered or not in the energy equation, respectively. Ratts & Raut [15]