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Analytical solution of dual-phase-lag heat conduction in a finite medium subjected to a moving heat source

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ABSTRACT

In the present study, the dual-phase-lag heat conduction model was employed to describe the thermal behavior of a square plate which is heated by a moving temporally non-Gaussian heat source. The Green's function approach was utilized to develop a general solution for the non-Fourier heat conduction equation in a two-dimensional finite medium. The temperature responses of the medium obtained from Fourier's law, hyperbolic heat conduction model and dual-phase-lag heat conduction model were compared to show the influences of the phase lag parameters. In addition, the effect of scanning speed of heat source on the temperature distribution was discussed. The vector diagrams of heat flux were illustrated to show its propagation.

1. Introduction

Heat conduction phenomenon is universal and the thermal treatment method is widely used in surface hardening, cladding, cutting, welding and other industrial processes. Moreover, it has been applied in modern medicine, such as laser surgery operations, thermal therapy, hyperthermia treatment and other remedy methods. The laser works as a reliable thermal source with superior controllability in the heat processes. The local surface of the substrate material, including biological tissues, metallic and nonmetallic materials, is suddenly irradiated by the laser beam. Consequently, the temperature of the inner region nearby the heated surface rises sharply [1]. Lots of investigations have been carried out to get deep insight into the thermal physical procedure.

Fourier's Law is the most commonly employed theory to describe the heat conduction process. Peterson et al. [2] presented the temperature distribution within a two-dimensional rectangular slab subjected to a time-varying and spatially decaying laser source. The in-plane effect during heat flux spreading was examined with the comparison between the analyses in one and two dimensions. Kumar et al. [3] developed a three-dimensional heat transfer model to illustrate the blown-powder laser cladding process based on the Fourier's Law. Tsai et al. [4] investigated the microscale heat conduction in an anisotropic thin-film which was subjected to an ultrafast laser and examined the nonlinear anisotropic effect of thermal properties on a thermal field. In addition, thermoelastic behavior has been considered by a number of researchers. Sun et al. [5–7] explored the vibration phenomenon and thermal stress in micro-beam, micro-plate and thin film during pulsed laser heating treatment. An analytical-numerical technique based on the Laplace transform method was utilized to solve the coupled thermoelastic problem. The effects of laser pulse energy absorption depth and different boundary conditions were discussed. Li et al. [8] obtained the analytical expression of the temperature increase of a rectangular μ-ILED device on an orthotropic substrate and the result was validated by finite element analysis.

The most commonly used theory to describe heat conduction process is the Fourier's law. The Fourier's law recommends that the heat flux is proportional to the temperature gradient, which implies that the heat wave propagates at an infinite speed. It puts up a good performance to construe the homeostasis problems, but can't fully model the transient ones [9]. To resolve this problem, many researchers have tried to make some modifications to Fourier's law by considering the non-Fourier effect. For instance, Cattnaeo [10] and Vernotte [11] developed a hyperbolic heat conduction model, hereinafter referred to as C-V model, by introducing a relaxation time, which allowed the temperature gradient to precede the heat flux. Later on, many researchers utilized the C-V model to investigate the heat transfer problems. For example, Mishra et al. [12–14] analyzed the combined mode conduction and radiation heat transfer in two dimensional square and concentric spherical enclosures with non-Fourier effect considered. The governing energy equation was solved with employment of the finite volume method and the volumetric radiative information was calculated. Kundu and Lee [15] presented an analysis of the heat conduction in the solar collector absorber plates based on both Fourier' Law and the C-V model. The results of the two methods were compared and the effect of the boundary conditions was studied. Daneshjou et al. [16] performed

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the non-Fourier heat transfer analysis of infinite two dimensional functionally hollow cylinders subjected to time-dependent heat source by utilizing an augmented state space method. Akwaboa et al. [17] acquired a numerical solution of the hyperbolic heat conduction equation in a thermal barrier coating structure under an imposed heat flux on the exterior of the substrate. Zhang and Shang [18] investigated the two dimensional non-Fourier heat conduction in a semi-infinite medium which was subjected to a laser beam on the local surface. Laplace and Hankel transforms was used to derive the analytical solution. Abbas [19] solved a problem on fractional order theory of thermoelasticity for a functional graded material. The governing equations of fractional order generalized thermoelasticity with one relaxation time for functionally graded materials were established.

In order to investigate the transient heat transfer process in microstructures, another heat conduction model called Dual-Phase-Lag model (DPL model), which allowed either the temperature gradient to precede heat flux or the heat flux to precede temperature gradient, was established by Tzou [20]. Later, Loh et al. [21] solved the heat transfer equation based on DPL model by using the asymptotic waveform evaluation method and compared the solution with that in Fourier's Law. Grysa et al. [22] presented a new approximate method to solve the governing equation with non-Fourier effect. Fu et al. [23] focused on a theoretical study of non-Fourier heat conduction in a sandwich panel with a cracked foam core and the thermal response of the plate was obtained. Kumar et al. [24] developed a three dimensional finite element based numerical model for femtosecond laser pulse heating using dual phase lag effect. Besides the engineering fields, DPL model was commonly employed in modern medicine treatment. Many researchers [25–29] explored the heating process of biological tissue in thermal therapy based on different heat conduction models, including Fourier' Law, C-V model and DPL model. Comparison among the results of the three theories suggested that the DPL model would lead to the same results as Fourier's Law in the case of $\tau_q = \tau_T$ under uniformly distributed initial condition.

Recently, the application of scanning heat sources keeps increasing in manufacturing process such as cladding, cutting and welding. Therefore, the thermal response of medium induced by a moving source is attracting more attentions. Among these, Malekzadeh and Shojaee [30] studied the dynamic response of functionally graded beams with temperature-dependent material properties which was irradiated by a moving heat source. The effect of different parameters on the thermoelastic behavior of functionally graded beams was considered. Shuja and Yilbas [31] developed a thermal stress analysis of laser multi-beam processing on a moving steel sheet. The finite element method was employed to obtain the temperature and stress distribution. Vergnaud et al. [32] studied the adaptation of the conjugate gradient method for the identification of heat flux densities provided by two mobile sources on a two-dimensional geometry. Sun et al. [33] formulated the thermoelastic behavior of a semi-infinite rod which was subjected to a temporally decaying moving laser pulse. The temperature, displacement, strain and stress were obtained by using Laplace transform method. Salimi et al. [34] derived an analytical solution to the practical industrial applications, in which the boundaries were subjected to a cooling or heating source moving in a finite domain. Flores and Zendejo [35] presented a numerical simulation of heat transfer involving a moving source with the use of the finite pointset method. Devesse et al. [36] tracked the isotherms on the surface of a semi-infinite work piece which is heated by a moving point source by utilizing an isotherm migration method. Al-khairy [37] attained an analytical solution of the one-dimensional hyperbolic heat conduction equation for a moving finite medium under the effect of a time-dependent heat source by employing the Laplace transform method.

Green's function method is useful in the solution of partial differential equations of mathematical physics. Much effort has been made to derive the corresponding Green's functions, which is the principal difficulty in the use of Green's function approach. For example, Sun et al. [38] investigated the heat conduction behavior of a bi-layered circular plate during pulsed laser heating by using Green's function method. Ozisik [39] showed how to obtain a Green's function by using variable separation technique for Fourier's model of heat conduction. However, it is more complicated to derive the Green's function for heat conduction equation with non-Fourier effect. In 1985, Frankel et al. [40] presented the Green's function approach for hyperbolic heat conduction in a one-dimensional medium. However, there are no reports about this approach for DPL model in 2D or 3D medium, which is a challengeable job.

The non-Fourier effect and moving heat source appear commonly in practical problems, especially in transient heat transfer or thermal process within non-metallic materials and biological tissues. The DPL model gives a great performance to describe heat conduction process with non-Fourier effect. Yet few literatures treat with the thermal response of medium subjected to a moving heat source by using the DPL model, because the thermal process would become more complicated under the combination of the DPL model and the moving source. To address this issue, the thermal behavior of a two-dimensional square plate subjected to a moving spot heat source is investigated by utilizing the DPL model in the present study. The heat source decays temporally in non-Gaussian form. The Green's function approach is utilized to develop a general solution for the DPL model equation in a two-dimensional finite medium and the analytical expression of temperature is obtained.

2. Heat conduction law

Tzou [20] introduces two phase lags to the Fourier's law, presenting the DPL model as

$$q(\vec{r}, t + \bar{\tau}_q) = -k\nabla T(\vec{r}, t + \bar{\tau}_T) \tag{1}$$

where q is the heat flux vector, T is the temperature, and k is the thermal conductivity. This heat conduction model allows either the temperature gradient to precede heat flux vector or the heat flux vector to precede temperature gradient. The parameters $\bar{\tau}_q$ and $\bar{\tau}_T$ are the phase lags of the heat flux vector and temperature gradient, respectively.

Taylor's series expansion of Eq. (1) up to the first-order terms in $\bar{\tau}_q$ and $\bar{\tau}_T$ leads to the following generalized heat conduction law:

$$\bar{q}(\vec{r}, t) + \bar{\tau}_q \frac{\partial \bar{q}(\vec{r}, t)}{\partial t} = -k\left[\nabla T(\vec{r}, t) + \bar{\tau}_T \frac{\partial}{\partial t}\nabla T(\vec{r}, t)\right] \tag{2}$$

Denote the components of $\bar{q}(\vec{r}, t)$ in x and y directions to be $\bar{q}_x$ and $\bar{q}_y$, respectively, then Eq. (2) can be written into

$$\begin{cases} \bar{q}_x + \tau_q \frac{\partial \bar{q}_x}{\partial t} = -k\left(\frac{\partial T}{\partial x} + \tau_T \frac{\partial^2 T}{\partial x \partial t}\right) \\ \bar{q}_y + \tau_q \frac{\partial \bar{q}_y}{\partial t} = -k\left(\frac{\partial T}{\partial y} + \tau_T \frac{\partial^2 T}{\partial y \partial t}\right) \end{cases} \tag{3}$$

The energy equation is [39].

$$-\nabla \cdot \bar{q}(\vec{r}, t) + Q(\vec{r}, t) = \rho C \frac{\partial T(\vec{r}, t)}{\partial t} \tag{4}$$

where $Q(\vec{r}, t)$ is the heat source term, which is generally specified as heat generation per unit time, per unit volume. ρ is the density of the material and C is the heat capacity.

Eqs. (2) and (4) yields the heat conduction equation of DPL model as

$$\nabla^2\left(T + \bar{\tau}_T \frac{\partial T}{\partial t}\right) + \frac{1}{k}\left(Q + \bar{\tau}_q \frac{\partial Q}{\partial t}\right) = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t} + \bar{\tau}_q \frac{\partial^2 T}{\partial t^2}\right) \tag{5}$$

where $\alpha = k/(\rho C)$ is the thermal diffusivity.

3. Mathematical formulation of the problem

Consider a rectangular plate which is subjected to a moving heat