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Modeling of stability control in a thin layer of solidifying melt on cylindrical surface

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ABSTRACT

Parametric oscillations of phase transition boundaries, which, unlike surface oscillations of liquid metal film flows described in literature, are shown as defined by an energy exchange between the electromagnetic regulator and the object. While the force influence was considered in film flows, here we consider the thermal influence that makes the frequency of the applied electromagnetic field irrelevant. It has to be rather high so that Joule heat dissipation could occur in a thin skin-layer near the boundary of phase transition (for implementation of the boundary control). Oscillations of phase transition boundaries reveal a low frequency; the system exhibits considerable inertness in thermodynamic relation. Thus, stabilization (suppression of oscillations) is possible at the expense of Joule heat dissipation, i.e. the stabilizing effect of an electromagnetic field is mediated and force actions, as it was demonstrated by our experiments, are rather small. The problem is of practical interest for protection of the metallurgical aggregate machines against high-temperature and chemically aggressive melt with artificial controlled thin layer of garnissage.

1. Introduction to the problem

Intensive development of modern equipment and new technologies, creation of highly efficient metallurgical units caused increasing attention of scientists and engineers to a heat and fluid flow control. This paper presents generalization and further development of the problem on stability control in a thin layer of solidifying melt that was partly considered in Refs. [1,2], with some examples of numerical computer simulations.

Casual oscillations of parameters (temperature, pressure, density, etc.) and possibility for effective impact on the processes in continua lead to substantial improvement of existing technological and metallurgical processes, as well as allow constructing the principally new ones. The increase in efficiency of metallurgical and electro-welding processes is often limited to overcoming the need of different magnetohydrodynamic instabilities, in particular, phase transition boundaries (fronts of solidification) of the melts. For example, instability of a thin layer of a solid phase of the metal (so called garnissage) intended for the walls' protection in metallurgical units against destruction doesn't allow effective usage of artificial garnissage [3,4], known in metallurgy mostly as a negative phenomenon [5]. Also, the instability of garnissage worsens the quality of melt due to its pollution with material of the metallurgical units' walls. In electric welding the control of phase transition boundaries allows significant improving of seam quality and reduction of the melting electrode metal consumption. Control of drops' formation processes and location of the melting boundaries gives the chance for essential improvement in the arc welding control [6,7].

Instability of a thin layer of cylindrical surface solidifying melt was studied with special focus on possibility for active suppression of instabilities by means of the automatic control. As a rule, a task of optimum control is set for the open systems with program control. Phase transition boundaries (e.g. solidification) are systems with distributed parameters, which behaviors in space and time are described by the partial differential equations. From the theory of automatic control in continua it is known that unstable linear object often can't be stabilized with just programmed influence, whereas the control by the principle of feedback, at which the level of operating influence is associated with perturbation of a system, is more effective.

Devices of automatic heat flux control [4] intended for stabilization of the phase transition boundaries of electro-conductive liquids can be constructed using high-frequency electromagnetic fields with the following action principle. A curvature of the phase transition boundary causes a change of current in the operating winding. An induced secondary current in a thin skin-layer almost coinciding with the phase

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transition boundary, owing to Joule heat dissipation, stabilizes the surface of the solidification front. A choice of control system parameters allows achieving suppression of practically any kind instability of phase transition boundaries. This allows stabilizing a garnissage and excluding metal's contact with the channel walls in metallurgical devices.

2. Statement by stability and stabilization of the phase transition

Stability of phase transition depends on many factors in various real physical conditions, e.g. multilayered walls of channels made by different materials of different thickness and physical properties, influence of convection, physical properties of melts, casual and regular perturbations, etc. Theoretical study of the boundary control is conducted by creation of physical and mathematical models of the corresponding physical situations. In case of the boundary form of phase transition controlled with the automatic systems [3,4], the impedance boundary conditions are stated on the interface connected to the regulator. As a rule, the regulator is connected to a considerable power source; the reverse influence of an object on the regulator is insignificant and can be neglected.

For unstable regimes the automatic control of the heat fluxes is used based on the phenomenon that perturbation of a boundary of phase transition leads to perturbation of magnetic field causing corresponding change of current in a winding. This current amplified in the operating circuit directs the secondary current in a thin skin-layer near the interfacial boundary. The Joule heat fluxes cause suppression of the corresponding perturbations. For each harmonics the impedance boundary condition is the following [4]:

$$\frac{dT_{m,k}}{d\vec{n}} = G_{m,k}T_{m,k}, \tag{1}$$

where $G_{m,k}$ may take any value as the coefficient of the feedback control system for suppression of the mode of oscillation, with the wave numbers m and k for cylindrical surface; $\vec{n}$ is the normal vector to the boundary surface. Boundary condition (1) created on the external surface of the channel by the automatic system of heat flux control has the following detail form:

$$\frac{dT}{d\vec{n}} = -\iint\limits_{-\infty}^{+\infty} G(\varphi, x, \eta, \xi)T(\eta, \xi, t)d\eta d\xi, \quad G(\varphi, x, \eta, \xi)$$
$$= A(t)\gamma_{\alpha,\beta}\gamma_{\alpha_1,\beta_1}e^{i(\alpha x+\alpha_1\xi)}e^{i(\beta\varphi+\beta_1\eta)}$$

where A(t) is a function of time (can be any positive or negative value), φ, x- cylindrical coordinates of the phase transition, η, ξ - corresponding integration coordinates for the action space of the control system; $\gamma_{\alpha,\beta}$ are densities of the wires in the control system. The structure of the available control systems for diverse physical situations, their detailed description as well as derivation and substantiation of the phenomenological impedance boundary conditions of the type (1) and the other similar ones are presented in the monograph [4].

The winding with individually adjustable feedback coefficient for each mode is used to control several harmonicas in a control system, providing high degree of resolution. Electromagnetic methods received the greatest practical applications among control methods for the boundaries of electro-conductive liquids. Electromagnetic fields are used in metallurgy and electric welding in various purposes: intensification of heat- and mass transfer (excitation of oscillations of the phase transition boundaries and management of melt circulation by means of the alternating fields and travelling-wave fields), for control of thermal instabilities in the processes of MHD-technologies, etc. [8–11]. The impact of high-frequency electromagnetic fields at the boundary control is thermal (Joule heat dissipation) [3,4,11]. Therefore, the frequency of electromagnetic field does not impact the solution of a task if connection with a control system is set by (1). In the first approximation, the skin-layer thickness can be neglected as small comparing to characteristic size of an ingot. Skin-layer is then assumed coinciding with a surface of solidification, which is actually an area of some thickness.

Oscillations of the boundary are shown low-frequency unlike the oscillations excited by ponderomotive forces [6,7], so that they have considerable thermodynamic inertness. Boundary stabilization happens due to Joule heat fluxes thereby the field action is not direct and the power expenses are rather small. The interfacial crystal-melt form significantly changes depending on solidification conditions [12]. Due to high diffusive mobility of atoms at the melting temperature and rather low value of surface energy, real interfacial surfaces have an essential curvature in the scales commensurable with sizes of the elementary cells. Roughness of interfacial crystal-liquid surface is defined by change of its free energy in the course of chaotic accession of atoms [13]. A curvature of boundaries is a consequence of a stability loss due to stresses and deformations [12]. Application of electromagnetic fields gives the chance to stabilize the deformation of phase boundaries that in certain cases has paramount importance.

3. Model of the solidification front in cylindrical channel

Investigation of the oscillations on the phase change boundaries from liquid to solid state starts from the Eigen oscillations. A cylindrical channel, whose wall can consist of any number of layers of various materials, is considered. A thin layer (film) of a solid phase is formed on the internal surface of a channel by solidifying of melt under special temperature condition. This solid film (garnissage) protects the walls against thermal and other destructive influences. On the other hand, it protects a melt from contamination with various impurities that is important for the special metallurgy in which exclusive requirements to purity of the melted and (or) transported liquid metal are imposed. Such problem was stated at the end of 1970th by the Academician V.M. Glushkov, director of the Institute of Cybernetics of Ukrainian Academy of Sciences, related to the development of the new jet type steel melting machines [3].

The scheme of the physical system including configuration of liquid-solid phases of the same material is presented in Fig. 1, where R_0- the radius of the area occupied by melt, r_0- thickness of a layer of a solid phase (garnissage), $r = R_0$- cylindrical surface of melt solidification

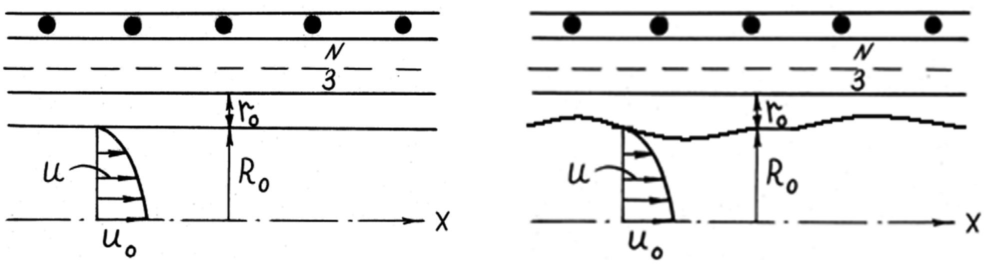

Fig. 1. Cylindrical configuration of liquid and solid phases with interfacial boundary: stable interface (to the left), deformed interface (to the right).