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Conjugate thermal creep flow in a thin microchannel

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ABSTRACT

In the present work, we analyze asymptotically and numerically the conjugate heat transfer between a rarified gas flow and the lower wall of a thin horizontal microchannel. The laminar motion of the gas is originated only by the thermal creep or transpiration effect on the lower wall of the microchannel. It is well known the need to impose, in general, a variable temperature regime at the lower wall to induce the transpiration effect. Usually, it can be reached by setting a linear temperature profile as a boundary condition. However, in our case, we prefer to avoid this simplification taking into account that in practical applications, the temperature profile at the lower wall can be unknown. This case can occur, for instance, in a heat sink or a similar device with a well defined heat dissipation rate. Under this physical configuration, we can assume then that the bottom or external face of this heat sink with finite thermal conductivity is exposed to a uniform heat flux. On the other hand, the upper wall of the microchannel is subjected to a prescribed condition. The above conditions are sufficient to consider the simultaneous or conjugate heat transfer analysis of the heat conduction equation for the heat sink and the mass, momentum and energy equations for the gaseous-phase. Resulting governing equations are written in dimensionless form, assuming that the Reynolds number associated with the characteristic velocity of the thermal creep and the aspect ratio of the microchannel, are both very small. The velocity and temperature profiles for the gas phase and the temperature profiles for the solid wall are predicted as functions of the involved dimensionless parameters and the main results confirm that the phenomenon of conjugate thermal creep exists whenever the temperature of the lower wall varies linearly or nonlinearly.

Nomenclature

Symbol	Definition
a'_0	Sound velocity evaluated at T'_0, $[m/s]$
A_{ws}	Constant for the temperature profile at the upper wall
C_p	Specific heat at constant pressure, $[J/(kg{\cdot}K)]$
C_v	Specific heat at constant volume, $[J/(kgK)]$
E'	Total energy by unit volume, $[J/m^3]$
e'	Internal energy per mass unity, $[J/kg]$
H	Microchannel width, $[m]$
h	Thickness of the heat sink, $[m]$
Kn	Knudsen number
$'$	Thermal conductivity of the fluid, $[W/(mK)]$
k'_w	Thermal conductivity of the lower wall or heat sink, $[W/(mK)]$
k_λ	Dimensionless coefficient given as $\sqrt{\pi/2}$
L	Microchannel length, $[m]$
Ma	Mach number
n	Exponent for the temperature profile of the upper wall
P'	Fluid pressure, $[Pa]$
P_0	Fluid pressure at the entrance of the microcahnnel, $[Pa]$
Pe_H	Pclet number
ΔP_c	Induced longitudinal pressure changes $[Pa]$
Pr	Prandtl number
q''	Heat flux, $[W/m^2]$
q''_s	Heat flux at the upper wall, $[W/m^2]$
R	specific gas constant
Re_H	Reynolds number
T'	Temperature, $[K]$
T_0	Fluid temperature at the entrance of the microchannel, $[K]$
T'_w	Wall temperature, $[K]$
T_{ws}	Temperature of the upper wall, $[K]$
$\Delta T'_{T,F}$	Characteristic transverse temperature change for the fluid, $[K]$
$\Delta T'_{T,F}$	Characteristic longitudinal temperature change for the fluid, $[K]$

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$\Delta T'_{T,w}$	characteristic transverse temperature change for the wall, $[K]$
$\Delta T'_{T,w}$	characteristic longitudinal temperature change for the wall, $[K]$
u'	Longitudinal velocity component, $[m/s]$
v'	Transverse velocity component, $[m/s]$
v'_m	$= \sqrt{2RT^{'}(x, 0)}$, the most probable molecular velocity at the lower wall, $[m/s]$
v'_m	$= \sqrt{2RT^{'}(x, H)}$, the most probable molecular velocity at the upper wall, $[m/s]$
u'_c	Characteristic longitudinal velocity, $[m/s]$
v'_c	Characteristic transverse velocity, $[m/s]$
u'_0	Velocity at the entrance of the microchannel, $[m/s]$
x, y	Longitudinal and transverse cartesian coordinates, respectively

Greek Letters

α_c	Conjugate heat transfer parameter
γ	Ratio of specific heats
γ_0	Dimensionless parameter
$\widehat{\gamma_0}$	Dimensionless parameter
Γ	Dimensionless sliding friction parameter
ε	$= H/L$, aspect ratio of the microchannel
ε_h	$= h/L$, aspect ratio of the heat sink
λ	Mean free path, $[m]$
μ'	Fluid dynamic viscosity, $[Pa{\cdot}s]$
μ'_0	Fluid dynamic viscosity at the entrance of the microchannel, $[Pa{\cdot}s]$
ξ_T	Jump temperature coefficient
ρ'	Fluid density, $[kg/m^3]$
ρ'_0	Fluid density at the entrance of the microchannel, $[kg/m^3]$
σ_p	Slip velocity coefficient
σ_T	Thermal slip coefficient

1. Introduction

Nowadays, it is well-known that for the study of gas microflows in microchannels and nanochannels, we can easily encounter at least the competing between the following effects: compressibility, viscous heating and rarefaction, [1]. The thermal creep effects studied here are included as rarefaction effects, considering the velocity slip and thermal jump conditions. The simultaneous participation of these effects can conduct in some cases to singular situations which are extremely difficult to address. One source of these difficulties is that rarefaction effects and non-equilibrium thermodynamic dominate at the inner surfaces of the microchannel and as a consequence, we must include some very specific slip conditions: in the slip regime, the Knudsen number, K_n, defined in the domain $0 < K_n < 0.1$, dictates the importance of these discontinuous effects, see Karniadakis et al. [1], Lockerby et al. [2], Roldughin [3]. Therefore, we adopt here a simplified point of view in order to elucidate only the role of one of them.

In the present case, we are particularly interested in understanding the conjugate thermal creep flow in a microchannel. In the following lines, we discuss the most relevant works related with the topic, taking into account that slip effects make possible to start the gas flow with tangential temperature gradients along the microchannel surface, i. e., the flow regime is controlled by a purely thermal creep effect, Roldughin [3]. However, in the present work, the tangential temperature gradients are unknown and must be determined as part of the problem. In fact, this thermal condition defines the conjugate problem. It does not prevent in any manner that the fluid starts creeping in the direction from cold toward hot, as was originally observed by Osborne Reynolds to define the thermal transpiration of gases through porous plates, Reynolds [4]. On the other hand, we consider only in this discussion, continuous approximations of the governing equations in contraposition with those cases where molecular simulations are carried out by solving the well-known Boltzmann equation, [5].

In this direction, Arkilic et al. [6] studied analytically and experimentally the gas flow with rarefaction along a microchannel by using a two-dimensional analysis of the Navier-Stokes equations with first-order slip-velocity boundary conditions. The analytical results based on a perturbation technique show that the streamwise mass flow coincides favorably with experimental measurements. An important study was also reported by Hadjiconstantinou and Simek [7], who developed the study of the convective heat transfer of a gaseous flow in micro and nanochannels, including the slip-flow regime when the walls of these devices are maintained to a uniform temperature.

Other peculiar cases that take into account microchannels of arbitrary shapes were considered by Zhu et al. [8], assuming a specific temperature gradient at the wall, avoiding in this manner, the conjugate formulation. However, these authors showed that the aspect ratio for the case of a rectangular microchannel has a remarkable effect on the dimensionless drag coefficient for a fixed K_n number. In this direction, Hossainpour and Khadem [9] explored also the fluid flow and heat transfer of gases circulating in microchannels taking into account different roughness shapes, just for the slip regime. They found that changes in roughness shapes from triangular to rectangular or trapezoidal profiles have an important impact on fluid flow and heat transfer characteristics.

In some works mentioned previously, second-order slip conditions have been taken into account; however, the multifactorial influence of Knudsen number, second-order slip conditions, creep flow, accomodation coefficients and hydrodynamically or thermally developing flow were considered simultaneously by van Rij et al. [10]. It should be noted that part of the previous works use numerical techniques for solving the corresponding governing equations. However, in some limiting cases, the use of analytical approximations can serve us to understand clearly different involved physical mechanisms. In this direction, Meolans and Graur [11] developed a very complete analysis of the thermal creep process in a rectangular microchannel imposing a uniform temperature gradient along the walls of the microchannel. An asymptotic result very relevant derived from the above work is the following: the induced pressure gradients along the microchannel are of order K_n^2, and therefore these gradients can be neglected from the Navier-Stokes equations, situation which was also confirmed numerically. Then the only mechanism to induce the motion of the fluid is dictated by the temperature gradients at the walls of the microchannel whether these gradients are known or unknown. Therefore, the above situation resembles or has a great similarity with the Couette flow: the fluid motion takes place with the sliding of one wall of the microchannel, while in our case the movement is achieved with the exclusive presence of temperature gradients on the walls of the microchannel. In both cases, the presence of external or induced pressure gradients are absent and this fact will be used in the following sections.

In addition, the induced pressure gradients by thermal creep velocity along the channel are negligible as was widely confirmed by Han [12]. The main result of those simulations shows that for small values of the Knudsen number, i. e., when the slip regime is valid, the induced pressure and the corresponding gradients can be neglected. These results can be seen from Figs. 9, 14 and 16 of the above work.

In recent years, many works have appeared in the specialized literature of the thermal transpiration considering different aspects such as extended Reynolds analogy for slip and transition flows, second-order corrections for slip conditions, entropy generation, variable thermophysical properties of the flow, entrance effects for the thermal creep, viscous dissipation effects, etc, [13–20]. However, the conjugate thermal creep flow has not been considered except in some exceptional cases. Here, we are specially interested in those conjugate heat transfer problems for which the simultaneous thermal interaction between the