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Analytical study of transient thermo-mechanical responses of dual-layer skin tissue with variable thermal material properties

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ABSTRACT

Biothermomechanics involves bioheat transfer, biomechanics, burn damage and physiology. Comprehension of biothermomechanics in living tissue is very important to clinical applications. In present work, the generalized thermoelastic theory without energy dissipation is used to investigate bioheat transfer and heat-induced mechanical response in bi-layered human skin with variable thermal material properties. The governing equations of skin tissue with temperature-dependent material properties are solved by Kirchhoff and Laplace transformation. The transient thermoelastic responses and thermal damage of the skin tissue are obtained and illustrated graphically. The influences of interface and variable thermal material parameters on the responses are also discussed.

1. Introduction

With advances in microwave, laser, focused ultrasound and radiofrequency, a lot of modern thermo-therapeutics have been widely used in clinical treatment. For example, the laser is focused on tumor by an objective lens for thermal therapy. One of the biggest challenges in thermal therapy is delivering the appropriate heat energy to the diseased tissue without affecting the healthy tissue. Thus a pressing need is to understand how the temperature/stress fields affect the kinetics of a thermal treatment. Van and Gybels [1] showed that the deformation due to heating and cooling can also produce pain sensation. Therefore, accurate predictions of heat and mechanical responses, as well as thermal damage in biological tissue are important for treating plan and designing new clinical heating systems.

Analysis of heat transfer in living biological tissue is a complicated physiological process due to the inherent characteristic in biological tissue, e.g. blood circulation, sweating, metabolic heat generation, and heat dissipation via hair or fur. In order to describe this complex phenomenon, Pennes [2] firstly established bioheat transfer equation to model the temperature profile in human forearm (Pennes model). Then other bioheat transfer models have been established to overcome the limitations in Pennes model, which does not consider the blood velocity field and the geometry of blood vessel [3–7]. Although these models are more complete and accurate than Pennes model, their complexity makes them quite complicated and difficult in practical application. On the other hand, Pennes model is simple with a small number of material parameters, thus many researchers have been attracted to improve this model [8].

It is noted that even a small change of heat-induced stress can suppress immune response, alter production of hormones and protein denaturation [9]. However, most studies mainly focus on the heat conduction [10–22], while the heat induced deformation is not considered. Based on the Pennes model, Shen et al. [23] studied the static thermo-mechanical responses of skin tissue at high temperature. Xu et al. [24,25] investigated the heat transfer, thermal damage and heat-induced stress of human skin. Kim et al. [26] analyzed the transient thermal-mechanical response of innocuous tactile stimulation induced by laser. Nevertheless, it can be found that the mechanical behavior has no effect on the temperature response in these studies.

It is known that in particular heat treatment conditions, i.e. high-power with short duration and cryogenic temperature, or heat conduction in media with non-homogeneous inner structure, heat travels at a finite speed [27–29]. Thus the classical uncoupled and coupled thermoelastic theories may be challenged in predicting accurate temperature and stress profiles in these conditions. To eliminate such paradox, the generalized coupled thermoelastic theories have been widely applied in investigating transient thermal shock problems. Lord and Shulman [30] developed the generalized thermoelastic theory with one relaxation time by using a wave-type heat conduction law to replace Fourier's heat conduction law. Green and Lindsay [31] introduced the temperature rate into the constitutive equations and developed a thermoelastic theory with two relaxation times. Green and Naghdi [32]

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Nomenclature

q_i	Components of heat flux vector (W/m^2)
k	Thermal conductivity (W/m K)
k^*	Material parameter of G-N theory
c_0	Constant specific heat at reference temperature (J/kg K)
T_0	Reference temperature, $T_0 = 310$K
e_{kk}	The cubical dilatation
T	Absolute temperature (K)
E	Activation energy (kJ/mol)
A	Frequency factor (s^{-1})
w_b	Blood perfusion rate (s^{-1})
T_b	Blood temperature (K)
Q_{met}	Metabolic heat generation (W/m^3)
R	Universal gas constant (J/mol K)
s	Laplace transformation parameter
h	The thickness of medium I
χ_1	Small quantity shows the influence of temperature (K^{-1})
L	The length of the model
x	Space coordinate
S	Entropy density of skin tissue
c_b	Specific heat of blood (J/kg K)
Ω	Tissue thermal damage parameter
c	Specific heat of skin tissue (J/kg K)
k_0^*	Constant material parameter at reference temperature
u_i	Components of displacement vector (m)
e_{ij}	Components of strain tensor
t	Time (s)

Greek symbols

θ	Temperature increment (K)
σ_{ij}	Components of stress tensor (pa)
δ_{ij}	Kronecker delta function
ρ_b	Blood mass density (kg/m^3)
ρ	Skin tissue mass density (kg/m^3)
ϑ	Kirchhoff transformation of θ
λandμ	Lame's constants (kg/ms^2)
α	Thermal expansion coefficient (K^{-1})$\gamma = (3\lambda + 2\mu)\alpha$

Subscript

i, j	Number of space domain

proposed a theory based on three types constitutive equations, which labeled as G-N I, II, III. When the theory is linearized, G-N I is equivalent to Fourier's heat conduction law; G-N II predicts heat propagating at a finite speed and involves no energy dissipation; G-N III includes a thermal damping term and thermal wave tends to diffusive with the increasing of damping coefficient. G-N II and G-N III model are typically presented to formulate a generalized thermoelasticity without energy dissipation and generalized thermoelasticity with energy dissipation, respectively, which predict the thermal propagating at a finite speed. Mitra et al. [33] carried out experiments on processed meat with different boundary conditions and observed the wave-like behavior in the heat transfer. The relaxation time of biological tissue is often estimated to be 10–30s and much larger than $10^{-12} - 10^{-14}$s in metal. Chandrasekharaiah [34] has reported that G-N II model was more applicable in some situations where relaxation time was larger than the time of interest.

In the above work, the material properties of biological tissue are often taken to be constant. However, at high temperature, tissue thermal properties and blood perfusion are no longer constant and change with temperature [35]. Lakhssassi et al. [36,37] and Tunc [38] considered variable material properties and studied various bioheat transfer problems in the context of Pennes model, in which the blood perfusion rate and thermal conductivity were linear functions of the temperature.

So far, the existing studies mainly focus on thermoelastic response with constant material parameters in living tissue, even if there are studies of variable material properties only limited to heat conduction [36–38]. Hardly any attempt is made to solve the thermoelastic coupling problem with temperature-dependent thermal material properties. One of the shortcomings in most bioheat transfer studies is modeling the skin tissue as a single layer with the same thermal material properties. This type of modeling may be challenged to predict precisely thermoelastic response of skin tissue whose properties are variable along the thickness.

Present work is devoted to investigate the transient thermoelastic responses of bi-layered skin tissue with temperature-dependent thermal material parameters in the context of generalized thermoelasticity without energy dissipation. The governing equations with variable thermal properties are solved by Kirchhoff and Laplace transformation. As a numerical example, the thermoelastic response of two-layered skin tissue under transient thermal shock is studied. The influences of interface and variable thermal material parameters on the temperature, displacement, stress and thermal damage are discussed and represented graphically.

2. Basic equations

It is assumed that skin tissue is uniform with linear, homogeneous and isotropic thermoelastic properties in the present work. Thus the thermoelastic equations in skin tissue with variable thermal properties based on the Pennes heat conduction model can be expressed as [23] (in the absence of any body forces):

$$q_i = -k(\theta)\theta_{,i} \tag{1}$$

$$q_{i,i} = -\rho T_0 \dot{S} + \rho_b w_b c_b (T_b - T) + Q_{met} \tag{2}$$

$$\sigma_{ij,j} = \rho \ddot{u}_i \tag{3}$$

$$\rho S = \gamma e_{kk} + \frac{\rho c(\theta)}{T_0}\theta \tag{4}$$

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \gamma\theta) \tag{5}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{6}$$

where $\theta = T - T_0$, $|\theta/T_0| << 1$and $e_{kk} = u_{x,x} + u_{y,y} + u_{z,z}$. The item $\rho_b w_b c_b (T_b - T)$ in the right side of Eq. (2) describes the heat conduction between blood and tissue. It is assumed $T_b = T_0$ in present paper. Besides, super-dot refers to the derivative with respect to time; comma followed by sub-index denotes the corresponding partial differentiation.

Substituting Eqs. (5) and (6) into Eq. (3), we can obtain

$$\rho \ddot{u}_i = (\lambda + 2\mu) u_{j,ij} + \mu u_{i,jj} - \gamma\theta_{,i} \tag{7}$$

Substituting Eqs. (1) and (4) into Eq. (2), the heat conduction equation based on Pennes model can be obtained:

$$(k(\theta)\theta_{,i})_{,i} + Q_{met} = \frac{\partial(\rho c(\theta)\theta)}{\partial t} + T_0 \gamma \dot{e}_{kk} + \rho_b w_b c_b \theta \tag{8}$$

If the Fourier's heat conduction law (Eq. (1)) is replaced by the following equation [32], which predicts heat propagating at a finite speed.

$$\dot{q}_i = -k^*(\theta)\theta_{,i} \tag{9}$$