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Laminar conjugate forced convection over a flat plate. Multiplicities and stability

Rizos N. Krikkis

Institute of Thermal Research, 2 Kanigos Str, PO Box 106 77, Athens, Greece

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Dedicated to Professor Dimitrios A. Kouremenos in Honor of his 80th Birthday.

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ABSTRACT

The multiple solutions of a conjugate heat transfer problem modeling laminar forced convection on one side of a flat plate and multi-boiling cooling on the other side has been numerically studied. The hydrodynamic and thermal boundary layers are treated by means of an integral technique coupled with axial conduction along the plate. The boiling process is modeled by a simple and generalized heat transfer coefficient valid for all three boiling regimes. Two problems with different boundary conditions have been considered and up to five solutions have been numerically calculated. The analysis reveals that the longitudinal conduction along the plate together with the nonlinear boiling curve, are the key processes responsible for the multiplicity features of the problem whereas the modified Brun number and the conduction-convection parameter significantly affect the solution. The boundary conditions have a profound effect on the bifurcation structure but they do not affect the stability properties since for both problems out of the five solutions three are stable and two unstable. Stability analysis shows that there exists a temperature distribution where all boiling modes may be present simultaneously along the plate.

1. Introduction

Conjugate hear transfer models require the simultaneous solution of the temperature field in an interacting solid-fluid system and consequently the heat exchange rate is not a priori known but rather part of the solution. On one hand this is definitely a more realistic description of a problem (i.e electronics cooling, heat exchangers, phase change heat transfer) but on the other hand this comes at the expense of simplicity since boundary conditions of prescribed temperature or heat flux can no longer be imposed on the solid-fluid interface, Perelman [1]. Among the first conjugate problems considered was the simultaneous heat transfer on both sides of a finite thickness wall, one side being heated or cooled by a convective flow and the other one maintained at a constant temperature. Luikov et al. [2] and Luikov [3] analyzed this problem both analytically and numerically. A conjugation criterion in the form of the Brun number was introduced as a measure of the wall to the thermal boundary layer resistance ratio. Asymptotic solutions for small and large Brun numbers were obtained by Payvar [4]. The problem has been revised by Pozzi and Luppo [5] and Pop and Ingham [6]. The analysis was extended by Karvinen [7] to include internal heat sources and by Prasad and Sankar [8] to include boundary layers with pressure gradient. Further insights were provided by Treviño et al. [9] by considering the effects of the axial conduction and by Vinnycky et al. [10] who employed a two dimensional model in both the fluid and solid sections. Mosaad and Ben-Nakhi [11] analyzed conjugate heat transfer from a vertical plate, with one side subjected to natural convection and the other one to forced convection. Recently analytical and semi-analytical solutions have been devised by Lindstedt and Karvinen [12] Shah and Jain [13] and Hajmohammadi et al. [14]. The effects of the plate thickness and the optimization of the plate cooling process under forced convection conditions has been carried out by Hajmohammadi et al. [15,16]. Moreover, interdisciplinary forced convection in the nano scale including electrodynamic and magnetic effects have been considered by Sheikholeslami et al. [17–19].

After the experimental work of Nukiyama [20], back in 1934, boiling on non-isothermal surfaces such as electrically heated wires [21–23], fins of various geometries [24–32], cylindrical and plane heaters [33–35] have received significant attention. The heat transfer process in wires and fins is usually modeled by balancing longitudinal axial conduction to the difference between the heat generated in the heating element and the heat dissipated to the boiling liquid. For a particular working fluid a boiling heat transfer

Abbreviations: CCP, Conduction-Convection Parameter.
E-mail address: rkrik@uth.gr.

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Nomenclature

a_j	constants in boiling heat transfer coefficient, Eq. (20) [−]
A	cross sectional area [m^2]
Br	modified Brun number [−]
c_p	specific heat capacity [J/(kgK)]
h	(H/H_{ref}) reduced heat transfer coefficient [−]
H	heat transfer coefficient [W/(m^{2}K)]
k	thermal conductivity [W/(mK)]
L	plate length [m]
P	perimetry [m]
Pe	$(u_\infty L/\alpha)$ Peclet number [−]
Pr	(ν/α) Prandtl number [−]
q	heat flux [W/m^2]
Q	reduced heat flux [−]
Re	$(u_\infty L/\nu)$ Reynolds number [−]
t	time [sec]
T	temperature [K]
X	distance along plate [m]
x	(X/L) dimensionless distance along plate [−]
y	transverse coordinate [m]
u	x-direction velocity [m/s]

Greek symbols

α	thermal diffusivity [m^2/s]
δ	hydrodynamic boundary layer thickness [m]
δ_T	thermal boundary layer thickness [m]
Δ	(δ_T/δ) ratio of thermal to hydrodynamic boundary layer thicknesses [−]
Θ	$[(T-T_{sat})/(T_{ref}-T_{sat})]$ dimensionless temperature [−]
Θ_{n-t}	knot temperature connecting nucleate and transition regimes [−]
Θ_{t-f}	knot temperature connecting transition and film regimes [−]
λ	eigenvalue [−]
ν	kinematic viscosity [m^2/s]
ξ	dimensionless thermal boundary layer thickness [−]
ρ	density [kg/m^3]
τ	$(\alpha t/L^2)$ dimensionless time [−]
ϕ	conduction-convection parameter (CCP) [−]

Subscripts

1	leading edge ($x = 0$)
∞	free stream flow
b	reference to boiling
L	trailing edge ($x = 1$)
ref	reference value
s	reference to steady state
w	reference to plate wall

Superscripts

(′)	derivative with respect to x

coefficient is used which is a function of the local superheat either determined experimentally or as piece-wise continuous power-law functions for each boiling regime, which facilitates analytical solutions for special values of the temperature superheat exponents. The non-linear coupling between conduction and boiling leads generally to three solutions, but in certain cases many more have been reported by Speetjens et al. [35] and Krikkis [36].

The aim of the present study is to analyze the multiplicity features of a common industrial application where a hot convective flow is used to excite the boiling modes of a saturated liquid. As a typical configuration, a flat plate is considered as a heat transfer interface between a stream of a heating fluid on the upper side, which provides the essential superheat, and a boiling liquid on the lower side. The convective part of the heat transfer mechanism was formulated in the framework of an integral approach. The reason is twofold. On one hand integral methods may yield quite adequate solutions for problems where an exact solution might be considerably more difficult and laborious to obtain, Kays and Crawford [37], Schlichting [38]. On the other hand when the model is cast in the form of a system of partial differential equations (i.e an infinite dimensional model) a discretization procedure (typically finite differences, finite elements) must be applied in order to transform the problem into a finite dimensional one (i.e described by ordinary differential equations) so it can be tractable by available bifurcation analysis tools, Seydel [39]. Furthermore, bifurcation and continuation analysis involve a demanding computational effort and it is usually required that the reduced order model is of relatively low order, Liu and Jacobsen [40]. The integral method is ideal for such an order reduction and at the same time accurate enough to avoid spurious bifurcations and erroneous predictions of stability. Taking into account that multiple solutions on non-isothermal surfaces have been well established both theoretically and experimentally it is reasonable to expect that the proposed model will exhibit similar multiplicities. Indeed up to five solutions may exist for certain values of the parameters. The solution structure is analyzed with sufficient bifurcation diagrams describing the multiplicity regions with the characteristic singular points.

2. Analysis

Consider a laminar convective boundary-layer flow along a flat plate of length L and thickness w. On the upper surface of the plate a hot fluid flows over with uniform velocity u_∞ and temperature T_∞. The lower surface of the plate is cooled by a pool boiling liquid at saturated temperature T_{sat}, as it is schematically depicted in Fig. 1. The physical and thermal properties of the fluid are assumed constant whereas viscous dissipation and axial conduction heat transfer are considered negligible. With the above simplifying assumptions, the energy balance in the laminar boundary layer may be expressed in the following integral form:

$$\frac{\partial}{\partial t}\int_0^{\delta_T}(T-T_\infty)dy+\frac{\partial}{\partial X}\int_0^{\delta_T}u(T-T_\infty)dy=-\alpha\frac{\partial T}{\partial y}\bigg|_{y=0}, \quad (1)$$

where α is the thermal diffusivity and δ_T is the thermal boundary layer thickness. The velocity profile in the boundary layer is assumed in the form of a third order polynomial as:

$$u/u_\infty=\frac{3}{2}(y/\delta)-\frac{1}{2}(y/\delta)^3, \quad 0\le y\le\delta, \quad (2)$$

which satisfies the hydrodynamic boundary conditions