International Journal of Thermal Sciences 111 (2017) 274–288

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

An analytical solution for magnetohydrodynamic Oldroyd-B nanofluid flow induced by a stretching sheet with heat generation/absorption

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ARTICLE INFO

Article history:
Received 9 March 2016
Received in revised form
10 August 2016
Accepted 11 August 2016

Keywords:
Three-dimensional flow
Oldroyd-B fluid
MHD
Nanoparticles
Heat generation/absorption
Convective boundary condition

ABSTRACT

This article provides an analytical investigation of magnetohydrodynamic (MHD) three-dimensional flow of an Oldroyd-B nanofluid in the presence of heat generation/absorption and convective surface boundary condition. Flow is induced by stretching surface considering the effects of Brownian motion and thermophoresis. The process of heat transfer is examined through the convective boundary condition. Oldroyd-B fluid is taken electrically conducting in the presence of a uniform applied magnetic field. A condition associated with nanoparticles mass flux at the surface is utilized. Problem formulation is made for boundary layer and low magnetic Reynolds number approximations. Suitable transformations are employed to construct the nonlinear ordinary differential equations. The strongly nonlinear differential equations are solved analytically through the optimal homotopy analysis method (OHAM). Effects of various interesting parameters on the temperature and nanoparticles concentration are studied and discussed. The local Nusselt number is also computed and analyzed. Our computations reveal that the temperature distribution has a direct relationship with Biot number and magnetic parameter.

1. Introduction

Viscoelastic liquids have attained special interest by recent investigators in view of wider industrial and engineering applications. Many of the industrial materials are characterized as viscoelastic liquids. Examples of such liquids include melts of polymers, biological solutions, paints, tars, glues, asphalts, colloids etc. The properties of all such liquids cannot be distinguished by the simple relation of stress and strain. Different constitutive relationships have been formulated for various rheological characteristics. For example, liquids exhibiting the relaxation of stress through constant strain are called Maxwell fluids. Maxwell fluids cannot predict the effects of memory. To predict the effects of memory and elastic, the Oldroyd-B fluid model has been proposed. Most of biological and polymeric liquids commonly exhibit the properties of memory and elastic. This model is frequently used to analyze small relaxation and retardation times. This model yields to a Maxwell fluid when retardation time vanishes. Further the Oldroyd-B model corresponds to a classical Newtonian fluid when both relaxation and retardation times are absent. Sajid et al. [1] firstly initiated boundary layer stagnation point flow of an Oldroyd-B model towards a moving sheet. They provided numerical solutions of dimensionless velocity distribution. Zheng et al. [2] reported unidirectional flow of an Oldroyd-B liquid generated by an accelerating plate. They presented closed form solution expressions of velocity and shear stress through Laplace transform technique. Shehzad et al. [3] analyzed the effects of thermophoretic particle deposition in three-dimensional flow of an Oldroyd-B fluid. They employed homotopic criteria to discuss the solutions of coupled velocity, temperature and concentration. Hayat et al. [4] studied mixed convection flow of an Oldroyd-B fluid over a radiative surface with double stratification and chemical reaction effects. Time-dependent three-dimensional flow of an Oldroyd-B fluid was numerically reported by Motsa et al. [5]. Hayat et al. [6] examined the influence of Cattaneo-Christov heat flux in magnetohydrodynamic flow of an Oldroyd-B fluid in the presence of homogenous and heterogenous reactions.

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http://dx.doi.org/10.1016/j.ijthermalsci.2016.08.009
1290-0729/

Nowadays, liquid cooling is a major issue due to recent advancements in modern technology because higher energy efficient heat transfer fluids are required to provide suitable conditions in industrial processes. Better liquid cooling can be obtained by submersion of tiny size nanoparticles into ordinary base liquids. Such fluids are known as nanoliquids and can be widely involved in different engineering processes like the food industry, refrigeration, waste heat recovery, air conditioning, automobile radiators etc. Transport behaviors of nanoliquids generally can be analyzed by the following two models given by Buongiorno [7] and Tiwari and Das [8]. Here we use the Buongiorno model [7] to study the convective heat transfer characteristics in nanofluids. This model determined that the homogeneous-flow models are in conflict with the experimental results and tend to underpredict the heat transfer coefficient of nanofluid. While the dispersion effect is totally negligible as a result of nanoparticle size. Thus, Buongiorno proposed an alternative model that ignores the shortcomings of homogeneous and dispersion models. He affirmed that the abnormal heat transfer appears due to particle migration in the fluid. Exploring the nanoparticle migration, he considered the seven slip mechanisms that can produce a parallel velocity between the nanoparticles and base fluid. These are inertia, thermophoresis, Brownian diffusion, diffusiophoresis, Magnus effect, fluid drainage and gravity. He concluded that, of these seven, only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. Based on such findings, he established a two-component four-equation nonhomogeneous equilibrium model for mass, momentum and heat transport in nanofluids. Keeping such facts in mind, Mustafa et al. [9] performed a study to examine the stagnation point flow of nanofluid towards a stretching surface. Makinde and Aziz [10] addressed boundary-layer flow of nanofluid due to a linear stretching surface with convective boundary condition. Sheikholeslami and Ganji [11] studied heat transfer characteristics of nanofluid flow between the parallel plates. Goodarzi et al. [12] examined mixed convection flow of nanofluid in a rectangular shallow cavity by considering two-phase mixture model. Malvandi and Ganji [13] explored mixed convection flow of nanofluid inside a vertical microchannel by utilizing modified Buongiorno's model. Natural convection flow of nanofluid over a vertical plate is investigated by Kuznetsov and Nield [14]. Hayat et al. [15] analyzed three-dimensional (3D) flow of second grade nanofluid due to an exponentially stretching surface in the presence of thermal radiation and heat source/sink. Hedayati et al. [16] discussed forced convection flow of nanofluid inside microchannels with asymmetric heating. Hayat et al. [17] studied three-dimensional (3D) boundary layer flow of Maxwell nanofluid in the presence of heat source/sink. Ellahi et al. [18] explored entropy generation in the flow of nanofluid with nanoparticles shape effects. Latiff et al. [19] examined unsteady forced bioconvection boundary layer flow of micropolar nanofluid past a stretching/shrinking sheet with multiple slip effects. Recently Uddin et al. [20] reported bioconvection flow of nanofluid over a moving plate in the presence of Stefan blowing and multiple slip effects.

Suspension of carrier fluid and magnetic nanoparticles is known as magnetonanofluid. The most important advantage of such liquids is that they are useful in controlling the heat transfer rate by implementation of an external magnetic field. Such characteristics of a magnetonanofluid are quite useful and interesting in thermal engineering, electronic packing and aerospace technology. Magneto nanofluids have been investigated by different researchers under various conditions and geometries. Thus, Turkyilmazoglu [21] provided an exact solution for magnetohydrodynamic (MHD) flow of nanofluid over a permeable stretching/shrinking surface with slip effect. Sheikholeslami et al. [22] studied magnetohydrodynamic (MHD) flow of nanofluid in a semi-porous channel. Malvandi and Ganji [23] examined forced convection slip flow of nanofluid inside a circular microchannel with a uniform magnetic field. MHD mixed convection flow of nanofluid inside a vertical annular pipe with asymmetric heating is reported by Malvandi et al. [24]. Gireesha et al. [25] discussed magnetohydrodynamic (MHD) three-dimensional flow of Eyring-Powell nanofluid over a convectively heated stretching surface with thermal radiation effect. Hayat et al. [26] analyzed magnetohydrodynamic (MHD) three-dimensional flow of couple stress nanofluid subject to nonlinear thermal radiation. Hayat et al. [27] also studied the squeezing flow of nanofluid over a porous stretching surface in the presence of time-dependent magnetic field.

Our aim is to model three-dimensional flow of an Oldroyd-B liquid over a moving surface through nanoparticles. Impact of heat generation accounts for the energy expression. Heat generation is a very important aspect in heat removal from nuclear fuel debris processes, storage of foodstuffs, disassociating fluids in packed-bed reactors, underground disposal of radioactive waste material, etc. Convective heat and zero nanoparticles mass flux conditions at the boundaries of the sheet are implemented. Implementation of such conditions leads to a more realistic physical problem. The available literature, mostly deals with constant surface temperature and nanoparticles concentration. Here we consider simultaneous effects of convective heat and zero nanoparticles mass flux conditions. The governing mathematical systems are solved via the optimal homotopy analysis method (OHAM) [28–37]. The results are presented graphically and discussed physically. Further convection is a heat transfer mechanism through a fluid in the presence of fluid bulk motion. Convection is categorized as free (or natural) and forced convection subject to how the fluid motion is initiated. In natural convection, any fluid motion is effected by natural means like the buoyancy effect, i.e. the rise of hot fluid and fall the cooler fluid. While in forced convection, the fluid is forced to flow past a surface by external means like a fan or pump.

2. Formulation

We consider steady three-dimensional (3D) flow of an incompressible Oldroyd-B nanofluid over a stretching surface subject to convective boundary condition. Flow is induced by a bidirectional stretching surface. The fluid is considered electrically conducting in the presence of a uniform magnetic field B_0 applied in the $z-$ direction. In addition, Hall and electric field effects are ignored. Induced magnetic field is not considered for a small magnetic Reynolds number [38–40]. Brownian motion, thermophoresis and heat generation/absorption effects are accounted. We adopt a Cartesian coordinate system in such a way that the $x-$ and $y-$ axes are taken along the stretching surface in the direction of motion and the $z-$ axis is normal to it. Let $U_w(x) = ax$ and $V_w(y) = by$ be the velocities of the stretching surface along the $x-$ and $y-$ directions. Here velocities of the stretching sheet vary linearly with distance from the origin. Such assumption is very realistic in several processes like the extrusion process in which material characteristics and in particular the elasticity of extruded sheet is being pulled out by a constant force. The temperature at the surface is managed by a convective heating phenomenon which is described by the heat transfer coefficient h_f and temperature of the hot fluid T_f under the surface. The governing boundary layer equations for three-dimensional (3D) flow of an Oldroyd-B nanofluid are [3,35]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$