International Journal of Thermal Sciences 111 (2017) 499–510

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

A perturbative thermal analysis for an electro-osmotic flow in a slit microchannel based on a Lubrication theory

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ARTICLE INFO

Article history:
Received 2 March 2016
Received in revised form
19 September 2016
Accepted 23 September 2016

Keywords:
Microchannels
Electro-osmotic flow
Non-isothermal
Joule heating

ABSTRACT

In this work, we develop a new thermal analysis for an electro-osmotic flow in a rectangular microchannel. The central idea is very simple: the Debye length that defines the length of the electrical double-layer depends on temperature *T*. Therefore, if exists any reason to include variable temperature effects, the above length should be utilized with caution because it appears in any electro-osmotic mathematical model. For instance, the presence of the Joule effect is a source that can generate important longitudinal temperature gradients along the microchannel and the isothermal hypothesis is no longer valid. In this manner, the Debye length is altered and as a consequence, new longitudinal temperature gradient terms appear into the resulting governing equations. These terms are enough to change the electric potential and the flow field. Taking into account the above comments, in the present study the momentum equations together with the energy and Poisson conservation equations are solved by using a regular perturbation technique. For this purpose, we introduce a dimensionless parameter α that measures the temperature deviations of a reference temperature. For practical cases, this parameter is small compared with unity and the theoretical predictions show; however, that for the used values of this parameter, the volumetric flow rate decreases in comparison with the isothermal case.

1. Introduction

Nowadays, the theoretical analysis of electroosmotic flows has allowed the development of important applications such as drug delivery, DNA analysis, biological/chemical agent detection sensors on microchips [1], etc. In general, electroosmotic flows are capable to induce fluid pumping and flow control using electric fields, eliminating the need for mechanical pumps or valves with moving components. Here, we are particularly interested in those physical aspects that can modify the estimation of the volumetric flow rates. For instance, the analysis of electro-osmotic flows in microchannels can be drastically altered by imposing non-isothermal conditions. In this direction, some analytical results for electro-osmotic flows in microchannels with different geometries can be found in Maynes and Webb [2], Su et al. [3] and Horiuchi and Dutta [4]. In parallel, Tang et al. [5] and Tang et al. [6] developed numerical studies under steady and transient conditions, determining the influence of the Joule effect on the electroosmotic flow. The numerical simulations reveal that the influence of the Joule effect is fundamental to understand simultaneously the electroosmotic flow and the mass species transport. Using triangle microchannels, similar results can be also found in Liao et al. [7]. Therefore, the above studies show the vital importance of the Joule heating effect. On the other hand, uniform or variable thermal properties of the electrolyte solution serves us to characterize more complex electroosmotic flow cases, where conjugate effects can be important as was previously studied by Sáanchez et al. [8], Escandón et al. [9] and Bautista et al. [10]. Other important cases where heat transfer processes have a special importance on the electroosmotic flow can be found also in Vocale et al. [11], Chakraborty et al. [12] and Escandón et al. [13]. Additional efforts including non-isothermal conditions, slip velocities, interfacial instabilities and concentration and mixed effects on electroosmotic flows circulating in microchannels with different geometries have been considered to clarify the importance of these additional effects. The most relevant contributions to appreciate the above aspects can be found in Matías et al. [14–16], Tandon and

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http://dx.doi.org/10.1016/j.ijthermalsci.2016.09.028

Kirby [17], Ge et al. [18] and Dey and Joo [19]. Special mention deserve the works of Bahga et al. [20], Ghosh and Chakraborty [21] and Yariv and Dorfman [22], where we can find special applications of electroosmotic flows affected also by axially varying surface conditions.

Although the previous specialized literature has reached an important place to clarify the role of the Joule effect; however, all previous contributions take into account that the ionic energy parameter [1] given by the relationship $\alpha_{ionic} = ez\zeta/k_BT$ is constant if and only if the temperature is uniform. Here, the symbols e, z, ζ, k_B and T denote the electron charge, the valence, the zeta potential, the Boltzmann constant and the temperature field of the electrolyte solution, respectively. However, in the present case of study and also in the references cited here, the temperature field of the electroosmotic flow is not uniform due to the presence of important longitudinal temperature gradients along the microchannel. These temperature gradients are originated not only by the presence of Joule heating effect but they are also caused by the pressure as will be shown later. In consequence, the above dimensionless parameter α_{ionic} varies with the temperature and also the well-known Debye length given by the relationship $\lambda_D = (2n_\infty z^2 e^2/\varepsilon k_B T)^{-1/2}$, where n_∞ and ε denote the ionic number concentration in the bulk solution and dielectric permittivity, respectively.

Following the previous comments, the aim of this work is to develop a new perturbative thermal analysis for a newtonian electroosmotic flow, taking into account that the presence of Joule heating effect and longitudinal pressure gradients can drastically alter, among others, the volumetric flow rate and the induced pressure field. For this purpose, we introduce a new dimensionless parameter α that defines the deviations of the temperature with respect to the isothermal case. For finite values of this parameter, we show that the momentum conservation equations have new terms which depend on temperature gradients and these terms are caused in turn by new pressure gradients which depend on the variable Debye length. However, in practical cases, this parameter α assumes very small values. Therefore, the governing equations are solved by applying a regular perturbation method for small values of this parameter. The perturbative predictions reveal that even for very small values of the parameter α, the volumetric flow rate together with the velocity, pressure, temperature and electric fields are accordingly modified.

2. Formulation and governing equations

In Fig. 1 we can appreciate a sketch of the physical model. Owing to this geometry, a 2D rectangular coordinate system (x,y) is adopted with the origin at the microchannel inlet and x-axis along the microchannel centerline. Here, we assume that a newtonian laminar fluid is circulating into the rectangular microchannel. The slit microchannel is basically characterized by the following geometrical dimensions: half height H and length L, such that $H \ll L$. In this manner, the above aspect ratio defines a long rectangular microchannel and the thickness of the walls is much smaller than any of the above geometrical lengths. We assume, in addition, the microchannel walls are very good conductors of heat generated by the Joule heating effect. Therefore, we accept also that the only mechanism for promoting the motion of the fluid is carried out by employing electroosmotic forces, which are induced by an electric field of strength E_0 in the longitudinal direction and given by $E_0 = \phi_0/L$, where ϕ_0 is the value of the imposed electric potential at the microchannel inlet. In addition, both ends of the microchannel are connected with two isothermal liquid reservoirs, founding at temperature T_0 and pressure P_0 [23,24]. The heat losses at the interval $0 \le x \le L$ for both lateral walls of the microchannel are controlled by a convective heat transfer process for which the heat transfer coefficient is known and given lines below. The above heat losses cause the temperature of the fluid can change in both longitudinal and transversal directions. Because the surface of the walls is negatively charged, we assume the existence of a thin electric double layer next to the walls and the ζ-potential between the charge surface and the electrolyte solution remains constant. Therefore, the net electric charge contained in the thin electric double layer is the primary reason for the electrokinetics effects, where positively charged ions can be driven by an external electric field. Together with the previous comments, we can add other well-known assumptions and hypothesis that complete the present mathematical formulation and they are basically the following [1]:

(a) the electric charge in the thin electric double layer obeys the well-known Boltzmann distribution. The above is a consequence for assuming a symmetric electrolyte of equal valence in equilibrium with the charged surface, (b) we assume also that the wall potential ζ is very small. In this manner, $e\zeta/k_BT_0 \ll 1$, considering that in practical cases $\zeta < 25$ mV. Therefore, the Debye-Hückel approximation for the Poisson-Boltzmann equation can be applied without any loss of generality; except for high values of the zeta potential. In this last case, the mathematical model develops here is no longer valid. In addition, the Debye-Hückel approximation remains still valid for those cases when temperature variations into the channel are important. This situation that corresponds to a non-isothermal electroosmotic flow, the linearization process for the Poisson-Boltzmann equation must take into account that the variable temperature effects, (c) the streaming potential induced by the flow is smaller than the external voltage, (d) the applied electric field is weak, i.e., $\phi_0/L \ll \zeta/H$. In this manner, the electric field is sligthly modified by a negligible amount at the double layer [25], (e) we assume $\lambda_D \ll H$, which means λ_D is a very thin layer, (f) the flow is laminar and the corresponding Reynolds number is very small compared with unity. In addition, because of $H \ll L$, which is typical for microfluidic systems driven by electrokinetic forces, the Lubrication approximation can be used to obtain the flow field, (g) in order to use the Boltzmann distribution, it is indispensable imposes that the ionic Peclet number is small, which is equivalent to assume equilibrium thermodynamics. The above can be appreciate from the Nernst-Planck species conservation equations that establish a species balance between ionic concentration gradients, electric and convective terms. In a dimensionless version of the above equations, the convective term can be neglected since it is multiplied by the ionic Peclet number and for small values of this parameter only prevails a balance between chemical and electric forces, (h) the fluid properties do not vary with temperature. However, this restriction can be eliminated as was widely studied by Bautista et al. [10] for the case of an electro-osmotic flow in a microchannel using the Phan-Thien and Tanner model, when the viscosity is a dependent function on temperature and finally (i) we assume that the total electric potential, Φ can be written as a linear superposition of the externally applied potential, ϕ and the electrical potential due to the thin electric double layer at the equilibrium state, ψ, i. e. $\Phi = \psi + \phi$, assumption which is only valid for a thin electric double layer ($\lambda_D \ll H$). Under this leading order consideration, therefore, the bulk of the fluid is completely free of charge (electroneutrality condition) and in the absence of imposed concentration gradients and under steady-state conditions, the external potential ϕ satisfies the Laplace equation.

Taking into account the previous assumptions and considerations, the electroosmotic flow equations are the mass conservation equation,