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Analysis of transient thermo-elastic problems using edge-based smoothed finite element method

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ABSTRACT

An edge-based smoothed finite element method (ES-FEM) is extended to deal with the transient thermo-elastic problems. For this edge-based smoothed finite element method, the problem domain is first discretized into a set of triangular elements, and the edge-based smoothing domains are further formed along the edges of the triangular meshes. In order to improve the accuracy, the ES-FEM utilizes the smoothed Galerkin weak form to obtain the discretized system equations in smoothing domains, in which the gradient field is obtained using a gradient smoothing operation. After applying these approaches, the numerical integration becomes a simple summation over each edge-based smoothing domain. The transient thermo-elastic problem is decoupled into two separate parts. At first, the temperature field is acquired by solving the transient heat transfer problem and it is then employed as an input for the mechanical problem to calculate the displacement and stress fields. Several numerical examples with different kinds of boundary conditions are investigated. It has been found that ES-FEM can achieve better accuracy and higher convergence in energy norm than the finite element method (FEM) when using the same triangular mesh.

1. Introduction

The analysis of transient thermo-elastic problems is of great importance in many practical engineering problems [1–3]. As it is quite difficult to find analytical solution for such problems with complex geometry and boundary conditions, numerical methods are widely employed to analyze these problems. In the past several decades, a number of numerical techniques have been developed and extended to solve thermo-elastic problems.

The finite element method (FEM) has been successfully utilized to solve heat transfer and thermo-elastic problems for a long time [4,5]. Chaudouet [6] applied the direct boundary integral equation (BIE) method to elastic analyses under thermal loading. Demirdzic [7] developed the finite volume (FV) method to analyze the behavior of continua under thermo-mechanical loads, which is based on the finite volume discretization. Recently, a node-based smoothed point interpolation method (NS-PIM) is applied by Wu [8,9] to analyze thermo-elastic problem, which can produce an upper bound solution. Ching [10] employed the meshless local Petrov–Galerkin (MLPG) method to investigate the transient thermo-mechanical response of two-dimensional solids. Singh [11–13] utilized the element free Galerkin (EFG) method to obtain the numerical solution of heat transfer and thermo-elastic fracture problems, in which the approximation function is constructed from a set of scattered nodes. Hematiyan [14] used the boundary element method (BEM) to analyze transient thermo-elastic problems, in which the problem domain is discretized using boundary elements.

Among all these methods, the FEM has been found to be a dominant numerical method. However, it has an inherent shortcoming known as the overly-stiff phenomenon, especially when linear triangular elements are used for two-dimensional (2-D) problems. To tackle this problem, Liu [15,16] has applied the smoothing technique in a number of meshing free and finite element settings. A generalized gradient smoothing technique has been utilized to establish the smoothed Galerkin weak form [17]. Liu et al. [18–20] have also proposed several ways (node-based, edge-based and cell-based) to construct the smoothing domains. It has been found that in the analysis of 2-D solid mechanics problems, the edge-based smoothed finite element method (ES-FEM) can give ultra-accurate solution especially when triangular meshes are used compared with the FEM.

In this work, the ES-FEM is further extended to solve transient thermo-elastic problems. At first, the computational domain is discretized into a set of triangular elements and the smoothing domains associated with the edges of the triangles are created

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Nomenclature

b	body force, N
e	emissivity
f	boundary traction, N
h_c	convection coefficient, W/(m^2 °C)
k	thermal conductivity, W/(m °C)
N	number of edges
n	unit outward normal component
q	heat flux, W/m^2
T_0	initial temperature, °C
T_w	known temperature, °C
T_∞	environmental temperature, °C

Greek symbols

Φ	shape functions for mechanical analysis
$\tilde{\sigma}, \tilde{\varepsilon}$	smoothed stress and strain
ε^E	elastic strain
ε^T	thermal strain
δ	delta operator
λ, μ, E, ν	Lamé's and elasticity constants
α	thermal expansion, 1/°C
β	Stefan–Boltzmann constant
ρ	density, kg/m^3
Ω	problem domain
Γ	global or local boundary

Subscripts and superscripts

i, j	tensor indices
k	smoothing domain for edge k

based on these triangular meshes. The discretized system equations are derived using the smoothed Galerkin weak form. Finally, both the transient temperature field and the stress field are calculated by the ES-FEM. Numerical examples with various kinds of boundary conditions are also presented to illustrate the validity of the ES-FEM for the transient thermo-elastic analysis through comparing the numerical results with those obtained by the FEM and the finite element commercial software ABAQUS.

2. Thermal analysis

2.1. Thermal governing equations and boundary conditions

It has been assumed that the material obeys Fourier's Law of heat conduction. Equations of the transient thermal analysis can be given by

$$\rho c \frac{\partial T(x,y,t)}{\partial t} = -\left(\frac{\partial R_x}{\partial x} + \frac{\partial R_y}{\partial y}\right) + Q(x,y,t) \tag{1}$$

$$R_x = -k_x\left(\frac{\partial T}{\partial x}\right) \tag{2}$$

$$R_y = -k_y\left(\frac{\partial T}{\partial y}\right) \tag{3}$$

where $T(x,y,t)$ is the temperature at time t, $Q(x,y,t)$ is the rate of internal heat generation, ρ is the density, c is the specific heat, k_x and k_y are the thermal conductivities, R_x and R_y are the rates of the heat flow in the x, y directions, respectively. The initial condition and thermal boundary conditions are simply stated here as

$$\text{Initial condition}: T = T_0 \tag{4}$$

$$\text{Dirichlet boundary}: T\big|_\Gamma = T_w \tag{5}$$

$$\text{Neumann boundary}: -k\frac{\partial T}{\partial n}\big|_\Gamma = q \tag{6}$$

$$\text{Robin boundary}: -k\frac{\partial T}{\partial n}\big|_\Gamma = h_c(T - T_\infty) \tag{7}$$

$$\text{Radiation boundary}: -k\frac{\partial T}{\partial n}\big|_\Gamma = \beta e\left(T^4 - T_\infty^4\right) \tag{8}$$

where T_0 is the initial temperature, T_w is the known temperature, T_∞ is the environmental temperature, q is the prescribed heat flux, h_c is the convection coefficient, n is the unit outward normal to the boundary. β is the Stefan–Boltzmann constant, e is the emissivity and Γ represents the boundary. It should be noted that Eq. (6) will change into the Adiabatic boundary when q is equal to zero.

2.2. Transient heat transfer analysis using ES-FEM

In the ES-FEM, the problem domain is also discretized using triangular elements as in the FEM. However, the stiffness matrices are calculated based on smoothing domains associated with the edges of the triangles, and the strain smoothing technique is used. The domain Ω is divided into M triangular elements with N edges, as shown in Fig. 1. After sequentially connecting two end points of the edge and the centroids of the triangle elements, smoothing domain of each edge is constructed. It can be clearly seen that, $\Omega_1 \cup \Omega_2 \cup \ldots \Omega_N = \Omega$ and $\Omega_i \cap \Omega_j = \phi$ ($i \neq j$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, N$).

In a triangular element, the temperature field T is interpolated using temperatures at the nodes of the element with linear shape functions which are the same as in the standard linear FEM.

$$T(x,y,t) = \sum_{i=1}^{3} \mathbf{H}_i \mathbf{T}_i(t) \tag{9}$$

where $\mathbf{T}_i(t)$ is the nodal temperature at node i and time t, $\mathbf{H}_i$ is the linear shape function. The temperature gradient $\mathbf{g}$ can be written by

$$\mathbf{g} = \mathbf{B}_T \mathbf{T}_d \tag{10}$$

$$\mathbf{B}_T = [\mathbf{B}_{T1}, \mathbf{B}_{T2}, \mathbf{B}_{T3}] \tag{11}$$

$$\mathbf{B}_{Ti} = \left[\mathbf{H}_{i,x}, \mathbf{H}_{i,y}\right]^{\mathrm{T}} \tag{12}$$

where $\mathbf{T}_d$ is the matrix of nodal temperature, $\mathbf{B}_T$ is the temperature gradient interpolation matrix. In order to overcome the overly-stiff phenomenon of the FEM, strain smoothing technique is utilized to soften the discrete system. In the kth smoothing domain Ω_k, which is formed by assembling two sub-domains Ω_{k1} and Ω_{k2} of two neighboring elements, the smoothed temperature gradient $\tilde{\mathbf{g}}_k$ is expressed as

$$\tilde{\mathbf{g}}_\mathbf{k} = \int_{\Omega_k} \mathbf{g}_\mathbf{k} \phi_k \mathrm{d}\Omega \tag{13}$$