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Time variation of combustion temperature and burning time of a single iron particle

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ABSTRACT

In the present study, combustion of a single iron particle is modeled by virtue of a novel thermophysical procedure. The temperature of iron particle during combustion is studied analytically and numerically. In the proposed model, the effect of thermal radiation from the external surface of burning particle, and alterations of density of iron particle with temperature are considered. Iron particle burns heterogeneously in air. Because of high thermal conductivity and micro size of iron particle, the Biot number is negligibly small, and the lumped system analysis can be utilized for the combustion modeling. The nonlinear energy equation resulted from modeling is solved by using homotopy perturbation method. The assumptions applied in the modeling are such that do not violate the actual combustion phenomenon. Also, the numerical solution of nonlinear differential equation is presented and compared with the analytical solution obtained from homotopy method. It is the first model presented for analyzing the combustion of single iron particle.

1. Introduction

Combustion of metallic materials is a challenging scientific subject that also has significant practical applications. Since the combustion of metallic particles is a high exothermic chemical reaction, use of them in the industrial operations is significant and efficient. Combustion of iron particle cloud experimentally investigated in some works. Iron is regarded as a non-volatile metallic fuel and the oxidation process takes place as a heterogeneous surface reaction. The major characteristic feature of iron combustion is that iron burns heterogeneously in air. It means that combustion reaction occurs at the iron particle surface and no flame is observed in the gaseous oxidizer phase. Iron particles do not evaporate during the combustion process and also combustion product (iron oxide) remains in the condensed phase. Because of high energy density of iron, the combustion of iron can be used as an alternative fuel in automobile engines and solid propellant rocket motors in the future. Recently, Bidabadi et al. [1] analytically investigated the distribution of iron dust particles through unburned zone across the flame propagation in a vertical duct by using homotopy analysis method. Also, Bidabadi et al. [2] theoretically proposed a mathematical modeling of velocity and number density profiles of particles across the flame propagation through a micro-iron dust cloud. Both studies focused on the dynamic behavior of iron particle clouds and various forces exerted on the particles in the preheat zone, prior to the flame zone. In addition, Bidabadi et al. studied the combustion of various type of particle cloud in several works [3–14]. Furthermore, Tang et al. [15] experimentally performed the so-called argon/helium test to determine the modes of particle combustion in iron dust flames. Experiments were accomplished in a reduced-gravity environment. Also, Tang et al. [16] experimentally studied the quenching and laminar flames propagating in fuel-rich suspension of iron dust in air in a reduced-gravity environment provided by a parabolic flight aircraft. Moreover, Sun et al. [17] measured the particle velocity and particle number density profiles across upward and downward flame propagating through iron particle clouds by using the high-speed photomicrographs. Sun et al. [18] experimentally examined the concentration profile of particles across a flame propagating through an iron particle cloud. Sun et al. [19] experimentally evaluated the temperature profile across the combustion zone propagating through an iron particle cloud. Also, Sun et al. [20] experimentally investigated the combustion behavior of iron particles suspended in air. In addition, Steinberg et al. [21–32] have carried out many experiments on the burning iron rods.

Homotopy is a mathematical technique for solving nonlinear differential equations [33,34]. Homotopy method has been used for

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solving various nonlinear differential equations raised in mathematical modeling of physical phenomena [35–43].

In the previous works, combustion of iron cloud has been experimentally investigated, or the dynamic behavior of iron particle in preheat zone has been studied.

This paper proposed a novel analytical approach to model the combustion of single iron particle burning in the gaseous oxidizing medium. By using this model, the variation of iron particle temperature during the combustion process as a function of time can be obtained. This model includes the thermal radiation effect and, also, variation of specific heat of iron particle by temperature.

This model applied for the combustion of iron particle for the first time, and can be improved in the future studies to include more complicated parameters that influence the circumstance of combustion. Because of nonlinearity of the governing differential equation, the homotopy perturbation method is utilized for solving the energy equation of iron particle.

2. Homotopy perturbation method

Consider the following nonlinear differential equation:

$$\mathscr{A}(u) - f(\mathbf{r}) = 0, \quad \mathbf{r} \in \Omega \tag{1}$$

with boundary conditions

$$\mathscr{B}\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad \mathbf{r} \in \Lambda \tag{2}$$

where $\mathscr{A}(u)$ is a general differential operator, $\mathscr{B}$ is a boundary operator, $f(\mathbf{r})$ is a known analytic function, and Λ is the boundary of domain Ω.

The operator $\mathscr{A}(u)$ can be divided to two parts $\mathscr{L}(u)$ and $\mathscr{N}(u)$, where $\mathscr{L}$ is linear, and $\mathscr{N}$ is nonlinear.

$$\mathscr{A}(u) = \mathscr{L}(u) + \mathscr{N}(u) \tag{3}$$

Hence Eq. (1) can be written as

$$\mathscr{L}(u) + \mathscr{N}(u) - f(\mathbf{r}) = 0, \quad \mathbf{r} \in \Omega \tag{4}$$

by means of homotopy technique [33,34], a homotopy function can be constructed, which satisfies the following equation:

$$\begin{aligned} \mathscr{H}(v,p) &= (1-p)[\mathscr{L}(v) - \mathscr{L}(u_0)] + p[\mathscr{A}(v) - f(\mathbf{r})] \\ &= 0, \quad p \in [0,1] \end{aligned} \tag{5}$$

or

$$\mathscr{H}(v,p) = \mathscr{L}(u) - \mathscr{L}(u_0) + p\mathscr{L}(u_0) + p[\mathscr{N}(v) - f(\mathbf{r})] = 0 \tag{6}$$

where

$$v(\mathbf{r},p): \quad \Omega \times [0,\ 1] \to R \tag{7}$$

where $\in [0,1]$ is an embedding parameter, and u_0 is the initial approximation of Eq. (1) which satisfies the boundary conditions. Obviously, we have

$$\mathscr{H}(v,0) = \mathscr{L}(v) - \mathscr{L}(u_0) = 0 \tag{8}$$

$$\mathscr{H}(v,1) = \mathscr{A}(v) - f(\mathbf{r}) = 0 \tag{9}$$

The process of changing p from zero to unity results in changing $v(\mathbf{r},p)$ from u_0 to $u(\mathbf{r})$. This is called deformation, and also $\mathscr{L}(v) - \mathscr{L}(u_0)$ and $\mathscr{A}(v) - f(\mathbf{r})$ are called homotopic in topology. If the embedding parameter $p \in [0,\ 1]$ is considered as a "small parameter", applying the classical perturbation technique, we can assume that the solution of Eq. (6) can be expressed as a power series in p, i.e.,

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \tag{10}$$

and setting $p = 1$ results in the approximate solution of Eq. (1) as

$$u = \lim_{p \to \infty} v = v_0 + v_1 + v_2 + \cdots \tag{11}$$

It is worth to note that the major advantage of He's homotopy perturbation method is that the perturbation equation can be freely constructed in many ways (therefore is problem dependent) by homotopy in topology and the initial approximation can also be freely selected.

3. Modeling of combustion of single iron particle

As the thermal diffusivity of substance is large and Biot number is very small ($\mathrm{Bi}_H \ll 0.1$), it is assumed that the particle is isothermal. In this state, a lumped system analysis is applicable. When this criterion is satisfied, the variation of temperature with location within the body will be slight and can be reasonably be approximated as being uniform. It means that the iron particle has a spatially uniform temperature, thus implying very large thermal conductivity of iron particle. Therefore, the temperature of particle is a function of time only, $T = T(t)$, and is not a function of radial coordinate, $T \neq T(r)$. The assumptions used in this modeling are:

1. The spherical iron particle burns in a quiescent, infinite ambient medium that contains only oxygen and an inert gas, such as nitrogen. There are no interactions with other particles, and the effects of forced convection are ignored.
2. Constant thermophysical properties for the iron particle and ambient gaseous oxidizer are used, except for the specific heat of the iron particle which varies by temperature.
3. The particle is of uniform temperature and radiates as a gray body to the surroundings without participation of the intervening medium.

First, iron particle is considered as a thermodynamic system, and by using of principle of conservation of energy (first law of thermodynamics), the energy balance equation for this particle can be written as

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} = \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_p \tag{12}$$

where $\dot{E}_{\text{in}}$ is the rate of energy entering the system which is owing to absorption of entire radiation incident on the particle surface from the surrounding surfaces, $\dot{E}_{\text{out}}$ is the rate of energy leaving the system by mechanisms of convection on the particle surface to the ambient oxidizer gas (air), and thermal radiation that emits from the outer surface of particle, $\dot{E}_{\text{gen}}$ is the rate of generation of energy inside the particle due to the combustion process and equals to the heat released from the chemical reaction, and $(\mathrm{d}E/\mathrm{d}t)_p$ is the rate of change in total energy of iron particle that is specified to the rate of change in the temperature of the particle.

$$\dot{E}_{\text{in}} = \alpha_s \dot{q}_{\text{incident}} = \alpha_s \sigma A_s T_{\text{surr}}^4 \tag{13}$$

where α_s is the absorptivity of the iron particle surface, $\dot{q}_{\text{incident}}$ is the flux of thermal radiation incident to the particle surface from the surrounding surfaces which enclose the particle,