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Determining true material constants of viscoplastic materials from rotational rheometer data

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ABSTRACT

We analyse the circular Couette flow of Herschel–Bulkley fluids to investigate the validity of the assumption that the rate of strain distributions across the gap share a common point. It is demonstrated that this is true only with fully-yielded Bingham -plastic flow. In other cases, e.g., in partially-yielded Bingham-plastic flow or fully-yielded Herschel–Bulkley flow, the common point for the fully-yielded Bingham case provides a good approximation for determining material constants only if the gap is sufficiently small. We also revisit the important issue of determining material properties of viscoplastic fluids by using "true values" for the rate of strain and demonstrate that the material properties can be very different from those obtained using "apparent' values for the rate of strain.

1. Introduction

The main objective in viscometry is to determine *objectively* fluid material constants independent of experimental and analysis errors. In principle, these material constants are determined by curve fitting using the measured values of stress and the rate of strain. However, in complex fluids, the rate of strain depends on both the velocity distribution and the material constants whose values of course are unknown and they are the objective of the analysis. Unfortunately, only in very few cases the velocity distribution is both known *a priori* and independent of the material constants. Traditionally then the velocity distribution is taken from the analytical solution of an *a priori* assumed constitutive model, such as the Newtonian or the power-law models. It turns out however that the rate of strain calculated using the predicted material constants is usually different from the initially assumed rate. This is because the initially assumed constitutive model for an arbitrary fluid is not necessarily the same as that determined by the experiments. For example, one can assume initially a Newtonian behavior to end up predicting a Herschel–Bulkley flow curve! Therefore, analyses where the rheology is assumed *a priori* can only yield "apparent" and not "true" material constants. As shown in [1] the introduced error in viscoplastic fluids can be significant. In the present work, by using data for typical viscoplastic fluids we demonstrate this fact and explain in detail of how "true" material parameters can be obtained.

In the following sections, we summarize already available analytical solutions for the circular Couette flow of Herschel–Bulkley materials and discuss the existence of common intersection points. Then, we analyze the flow in the general case and propose a systematic method for the determination of the material constants of Herschel–Bulkley fluids. We focus on an interesting approach proposed by Schummer and Worthoff [3] where in viscometric flows flow curves can intersect at a common point within the rheometer whose location is independent of the rheology of the fluids. Schummer and Worthoff [3] demonstrated this concept for a number of flows and found approximate locations where the resulting experimental error is minimum. Obviously, this is a profound result because one can experimentally get "true" material constants without iteration or other corrections. We investigate this concept for the case of Herschel–Bulkley fluids in a rotational rheometer whose basic flow is represented by the classical circular-Couette flow. The general problem of the rate of strain being a function of the material constants can be resolved by proper iteration between the experimental data and the predicted model parameters [2]. Finally, the method is applied to available data for a cosmetic emulsion and a coal-water mixture. To our knowledge, this is the first time that comparisons between true and apparent values of the rheological parameters are made. For the particular experimental data, the relative errors in the consistency index, the power-law exponent, and the yield stress are found to be high, moderate, and very small, respectively.

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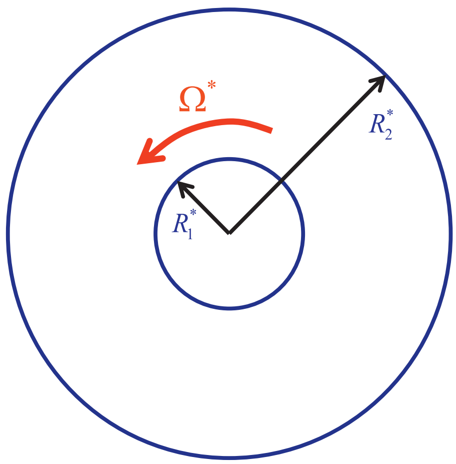

Fig. 1. Schematic of the geometry of the flow.

2. Theoretical framework

Let us consider the steady flow of a viscoplastic material between two co-axial, infinitely long cylinders of radii R_1^* and R_2^*, where $R_2^* > R_1^*$. It should be noted that throughout this work the stars denote dimensional quantities. Symbols without stars will denote dimensionless variables and parameters. The inner cylinder is rotating at a constant speed Ω^* while the outer cylinder is fixed, as illustrated in Fig. 1. The solution of this flow can be found in the literature (see, e.g., [1]). It is thus conveniently summarized here in order to provide the theoretical basis for the determination of "real" material constants and to illustrate the range of validity of the assumption that there is a common point independent of the fluid rheology.

Under the assumption of axisymmetric flow only in the azimuthal direction, the conservation of linear momentum for any fluid yields

$$\tau_{r\theta}^* = \frac{c^*}{r^{*2}} \tag{1}$$

and the rate of strain is given by

$$\dot{\gamma}^* = r^* \left| \frac{d}{dr^*}\left(\frac{u_\theta^*}{r^*}\right) \right| = -r^* \frac{d}{dr^*}\left(\frac{u_\theta^*}{r^*}\right) \tag{2}$$

The constitutive equation for a Herschel–Bulkley fluid may be written in scalar form as follows:

$$\begin{cases} \dot{\gamma}^* = 0, & \tau^* \le \tau_0^* \\ \tau^* = \tau_0^* + k^* \dot{\gamma}^{*n}, & \tau^* > \tau_0^* \end{cases} \tag{3}$$

where $\tau^* = |\tau_{r\theta}^*|$, τ_0^* is the yield stress, n is the power-law exponent, and k^* is the consistency index. The above model is a combination of the Bingham-plastic model ($n = 1$) and the power-law model ($\tau_0^* = 0$). The Newtonian fluid corresponds to $n = 1$ and $\tau_0^* = 0$.

In what follows, we will work with non-dimensionalized equations. The dimensionless variables are defined by

$$u_\theta \equiv \frac{u_\theta^*}{\Omega^* R_1^*}, \quad r \equiv \frac{r^*}{R_1^*}, \quad \dot{\gamma} \equiv \frac{\dot{\gamma}^*}{\Omega^*}, \quad \tau \equiv \frac{\tau^*}{\tau_0^*} \tag{4}$$

With the above scalings, the dimensionless form of the constitutive equation (3) is

$$\begin{cases} \dot{\gamma} = 0, & \tau \le 1 \\ \tau = 1 + \frac{1}{Bn}\dot{\gamma}^n, & \tau > 1 \end{cases} \tag{5}$$

where

$$Bn \equiv \frac{\tau_0^*}{k^* \Omega^{*n}} \tag{6}$$

is the Bingham number. Combining Eqs. (1) and (5), the non-dimensional rate of strain in the yielded regime ($\tau > 1$) is given by

$$\dot{\gamma} = Bn^{1/n}\left(\frac{c}{r^2} - 1\right)^{1/n} \tag{7}$$

where $c \equiv c^*/R_1^{*2}\tau_0^*$. To obtain the velocity u_θ^* one simply needs to integrate and apply the boundary conditions. Below a certain critical Bingham number, Bn_{crit}, the fluid is yielded everywhere in the gap $1 \le r \le R_2$. In this case the boundary conditions are $u_\theta(1) = 1$ and $u_\theta(R_2) = 0$ (no-slip boundary conditions), which lead to the following expression for the velocity

$$u_\theta(r) = r\left[1 - Bn^{1/n} \int_1^r \frac{1}{\xi}\left(\frac{c}{\xi^2} - 1\right)^{1/n} d\xi\right], \quad 1 \le r \le R_2 \tag{8}$$

where the constant c is calculated by demanding that

$$Bn^{1/n} \int_1^{R_2} \frac{1}{\xi}\left(\frac{c}{\xi^2} - 1\right)^{1/n} d\xi = 1 \tag{9}$$

Above the critical value Bn_{crit}, the fluid is yielded only partially in the range $1 < r < r_0$, where $r_0 < R_2$ is the radial distance from the inner cylinder to the point where $\tau = |\tau_{r\theta}| = 1$. From Eq. (7) it is deduced that

$$c = r_0^2 \tag{10}$$

and therefore

$$\dot{\gamma} = \begin{cases} Bn^{1/n}\left(\frac{r_0^2}{r^2} - 1\right)^{1/n}, & 1 \le r \le r_0 \\ 0, & r_0 \le r \le R_2 \end{cases} \tag{11}$$

and

$$u_\theta(r) = \begin{cases} r\left[1 - Bn^{1/n} \int_1^r \frac{1}{\xi}\left(\frac{r_0^2}{\xi^2} - 1\right)^{1/n} d\xi\right], & 1 \le r \le r_0 \\ 0, & r_0 \le r \le R_2 \end{cases} \tag{12}$$

where r_0 is a root of

$$Bn^{1/n} \int_1^{r_0} \frac{1}{\xi}\left(\frac{r_0^2}{\xi^2} - 1\right)^{1/n} d\xi = 1 \tag{13}$$

3. Special solutions when $1/n$ is an integer

For selected values of n, i.e. $n = 1$, $1/2$, $1/3$ etc., the equations used in the general case can be integrated analytically [1].

3.1. The Bingham plastic case ($n = 1$)

For a Bingham plastic ($n = 1$) it turns out that for $Bn > Bn_{crit}$,

$$u_\theta(r) = r\left[1 - Bn\left\{\frac{r_0^2}{2}\left(1 - \frac{1}{r^2}\right) - \ln r\right\}\right], \quad 1 \le r \le r_0 \tag{14}$$

and

$$\dot{\gamma} = Bn\left(\frac{r_0^2}{r^2} - 1\right), \quad 1 \le r \le r_0 \tag{15}$$

where r_0 is a root of

$$Bn = \frac{2}{r_0^2 - 2\ln r_0 - 1} \tag{16}$$

Obviously, the critical Bingham number Bn_{crit} above which the flow is partially yielded is:

$$Bn_{crit} = \frac{2}{R_2^2 - 2\ln R_2 - 1} \tag{17}$$

It is instructive to plot Bn_{crit} versus the outer radius R_2, as in Fig. 2. The critical Bingham number increases exponentially with the rheometer gap ($R_2 - 1$). When the dimensionless gap is 0.01 ($R_2 = 1.01$) the critical Bingham number is so high ($Bn_{crit} = 10033$) that one can safely assume that the flow is fully yielded for all non-exotic viscoplastic materials. However, if the gap is big the critical Bingham number is low and the possibility of having partially yielded flow cannot be excluded. For example, $Bn_{crit} = 103.2$ and 26.54 when $R_2 = 1.1$ and 1.2, respectively.