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Free-surface flow of a viscoplastic fluid during the filling of a planar channel

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ABSTRACT

The non-Newtonian fluid flow with a free surface occurring during a plane channel filling in the gravity field has been simulated numerically. The mathematical statement of the problem is formulated on the basis of the motion equations, continuity equation, and natural boundary conditions on the free surface with an application of the Herschel-Bulkley rheological model. A traditional mathematical model singularity on the three-phase contact line is eliminated using a slip condition. A numerical algorithm based on the finite-difference method is developed for solving the problem. Regularization of the rheological equation has been carried out using a shock-capturing method for the flow with unyielded regions. A parametric investigation of the flow kinematics and free surface behavior in terms of the governing parameters has been implemented. The flow structures distinguished by the presence of unyielded regions have been demonstrated depending on relation of the viscous, gravity, and plastic forces in the flow.

1. Introduction

Filling of tanks with a fluid is one of the main stages of injection molding which finds a widespread application in producing articles from polymer compositions and metals. In general, polymeric fluid flows during filling process are characterized by the complex rheological behavior, non-isothermality, chemical transformations, and by the presence of a free surface [1]. Mathematical model, which considers all these factors and, therefore, contains a large number of constitutive parameters, makes it more complicated to study a process and to predict a flow pattern. In this regard, when investigating the effect of one or other factor on process characteristics, it is worthy to confine mathematical model to take into account a certain factor. Such an approach promotes a fundamental understanding of the functional relations occurred using less constitutive parameters and makes it possible to realize if a more comprehensive model is needed.

Depending on processing conditions, rheology of liquid polymer compositions can vary from a typical Newtonian fluid to the medium exhibiting nonlinear viscous and viscoplastic properties [2]. Filled polymer compounds represent the structured materials with a specific feature of yield stress [3–5]. Yield stress is one of the basic characteristics of viscoplastic fluids which is still a matter of arguments in rheology [4]. Nevertheless, yield stress is definitely a favorable notion as regards to a structured fluid flow modeling. According to the definition of yield stress, viscoplastic fluid flow can be split into distinct regions. In the regions where the local shear stress level is less than yield stress, the fluid exhibits properties of a solid body. These flow zones are called the unyielded regions. Where the local shear stress level exceeds yield stress, the medium flows like a fluid with corresponding apparent viscosity. These regions are separated from one another by surfaces defined as positions of the points where the local shear stress level is equal to yield stress. The presence of two distinct regions is a significant factor for a flow structure development as it affects the physical characteristics of molded articles. The properties of the medium are expected to remain constant in the unyielded regions and different from those in the viscous flow zones.

Since the second half of the last century, many investigations have been devoted to the filling of plane and axisymmetric tanks using a Newtonian fluid behavior approach, taking into account a dissipative heating and viscosity temperature dependence. In [6–8], there is a detailed discussion of the problem state up to the present day. In the framework of a mathematical simulation, the approximate and numerical methods are used to solve the stated problems. It is noted that two types of zones can be distinguished during a channel filling: one-dimensional flow zones located far away from a free surface and a fountain flow near free surface. The medium motion in the vicinity of the interface of two immiscible streams accompanied with displacement of one fluid with another is generally referred to as a fountain flow [9].

Physicochemical hydromechanics of non-Newtonian fluids also demonstrates a rapid development. Although, a significant amount of works is done on the simulation of non-Newtonian medium flow with free surface, only few of them are devoted to a mold filling study [10–19]. Moreover, in these studies, very little data on the stream

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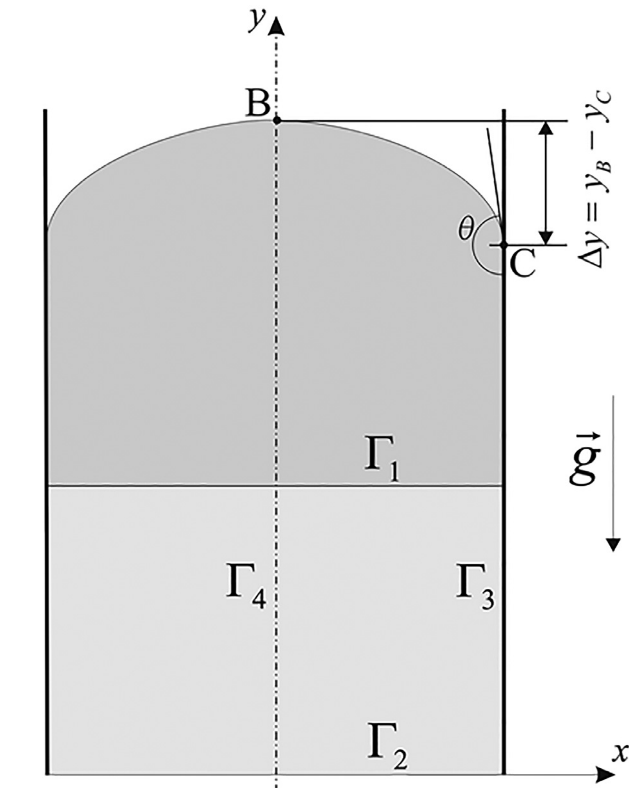

Fig. 1. Solution domain.

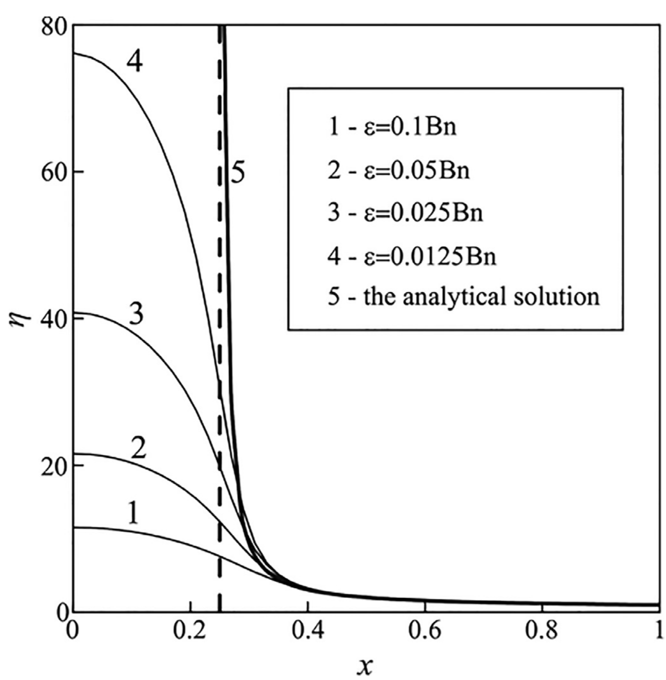

Fig. 2. Viscosity distribution in the cross section of $y = 4$ in a one-dimensional flow zone (Bn = 1, m = 0.8, W = 5, dashed line indicates the boundary of unyielded region).

Table 1
Effect of the regularization parameter on the flow characteristics.

ε	0.1Bn	0.05Bn	0.025Bn	0.0125Bn
V_2 / E_1	1.31063/0.88	1.30137/0.17	1.29657/0.20	1.29472/0.34
h / E_2	0.257/1.98	0.256/1.59	0.254/0.79	0.251/0.40

structure, flow kinematics, and free surface evolution are presented in respect to the rheological parameters.

Application of modern computational technologies provides an opportunity to implement adequate models and to predict a free surface evolution and the fountain flow details more precisely. Numerical modeling of unsteady flows with free surfaces is more complicated due to a necessity to determine both the solution domain and dynamics of a three-phase (gas-liquid-solid body) contact line. The problem of a three-phase contact line dynamics and the appropriate solution methods are discussed in [20–22]. In the classical model, singularity is eliminated using a slip condition on the solid wall in the vicinity of contact line. In some models, slip mechanisms are used as boundary conditions when a gas layer is formed on the solid surface due to the following reasons: air entrainment in the free surface front [23–25], limiting adhesion stress existence [26], wall roughness [27], relaxation properties of the interface in the vicinity of contact line [28], etc. A fairly complete review of the contact line dynamics model is presented in [29].

Consideration of a non-Newtonian fluid behavior requires additional efforts to implement computational technologies successfully. On the one hand, appearance of the additional nonlinear terms in the constitutive equations and boundary conditions as well as an effective viscosity variation in the solution domain significantly affect both the conditions and convergence rate of computational algorithms. On the other hand, a distinctive feature of the problems of viscoplastic flows is the necessity to determine the solutions in the domains with undetermined boundary of unyielded regions. This circumstance makes realization of the effective numerical methods more laborious. The main difficulty in a numerical simulation of viscoplastic medium flow is related to a singularity of rheological equations and impossibility to calculate the stress in the regions with zero strain rate. Regularization methods are widely used to cope with this problem [30,31]. Regularization represents the approximation of non-differentiable constitutive equations by a smooth function intended to solve the problem of the flow of fluid with nonlinear apparent viscosity. For regularized models, the term «unyielded region» is undefined, and the presence of such zones is introduced by a negligible strain rate condition or by the von Mises yield criterion. In this regard, the boundaries of unyielded regions in the flow are determined by stress isosurface with the value equal to yield stress. Mathematical aspects of convergence and estimation of the errors for various regularization methods are discussed in [32]. It is noteworthy that regularization methods can cause the essential errors in the unyielded region localization [32,33]. Therefore, application of a regularization method requires additional testing of the calculation results. Moreover, the unregularized methods are developed for viscoplastic flow calculations which indicate the unyielded regions more precisely, for example, the augmented Lagrangian methods. The current trends in numerical simulation of viscoplastic flows are presented in details in review [34].

The aim of this work is to investigate an effect of the plastic properties of liquid medium on the flow structure formation and flow kinematics during a plane channel filling on the basis of the Bingham-Shvedov and Herschel-Bulkley rheological equations [35–37].

2. Formulation of the problem

A plane vertical channel filling with an incompressible Herschel–Bulkley fluid is considered. The flow direction is opposite to that of the gravity force. The process is described by the motion and continuity equations written using dimensionless variables as follows:

$$\mathrm{Re}\left(\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y},$$
$$\mathrm{Re}\left(\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \mathrm{W}, \qquad (1)$$