Journal of Non-Newtonian Fluid Mechanics 231 (2016) 56–67

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

Transiences in rotational electro-hydrodynamics microflows of a viscoelastic fluid under electrical double layer phenomena

P Abhimanyu[a], P Kaushik[a], Pranab Kumar Mondal[a,b], Suman Chakraborty[a,*]

[a] *Department of Mechanical Engineering, Indian Institute of Technology Kharagpur, Kharagpur, 721302, India*
[b] *Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati, 781039, India*

ARTICLE INFO

Article history:
Received 11 November 2015
Revised 9 March 2016
Accepted 12 March 2016
Available online 17 March 2016

Keywords:
Oldroyd-B Fluid
Rotational fluidics
Electro-osmotic flow
Transient analysis
EDL phenomena

ABSTRACT

We analyze the transiences in rotational electro-osmotic flow of a viscoelastic fluid in a microfluidic channel. We here use Oldroyd-B equations to model the viscoelastic fluid. We bring out the rotation induced complex flow dynamics during the transience, leading to possible augmentations in mixing, as modulated by the modifications in the force distribution in the flow-field. We attribute alterations in these forces as a combined consequence of the modifications in stress under the simultaneous action of rotation and electrical forcing and its interplay with the elastic stress originating from the viscoelastic effects. We also highlight on the volumetric transport characteristics as dictated by the underlying dynamical conditions. The inferences obtained from the present study may bear significant consequences in the design of various microfluidic devices, which are commonly used for the transportation of bio-fluids such as blood.

1. Introduction

Rotational fluidics addresses the flow physics in confinements under angular motion [1–4]. Such considerations are important for the design and analysis of portable devices (for example, lab-on-a-CD) tailored for a plethora of applications ranging from augmented microfluidic mixing [5–13] to medical diagnostics [5,14,15]. While several studies have been reported on the fluid dynamics in rotationally actuated devices [16–18], most of the pertinent investigations have considered simple constitutive models [19–22] for capturing the essential physics of interest. In addition, influences of other actuating parameters (such as electric field) have not been commonly considered towards interrogating the possibilities of realizing augmented functionalities of the rotationally actuated devices. Such a paradigm, however, could be potentially possible, by considering the interplays of the various forcing parameters. The underlying physical considerations are expected to be by no means trivial, because of plausible complicated non-linear interaction between the various forcing parameters (mediated by the inertial effects) as well as the flow rheology. The issue of complex rheological characteristics becomes even more relevant, when one intend to employ typical biological fluids like blood (for example, for medical diagnostic applications) [5,14,23], in rotationally actuated microfluidic devices.

We would like to mention here that although the viscoelastic model is mathematically convoluted, it captures the intricate rheological effects of many biological fluids on the flow dynamics quite efficiently. Also it should be highlighted in this context here that the lab-on-a CD based microfluidic devices that are commonly used for the transportation and analysis of DNA solutions, blood and bio-fluids are intrinsically actuated by rotational means. Such devices require a delicate compromise between mixing and throughput [5,15,24,25]. Accounting above all issues, an attempt towards addressing the underlying dynamical behavior of a viscoelastic fluid in a rotation induced flow environment as modulated by the electrical double layer effect could be an interesting proposition, mainly attributed to the rich physical interplay of various spatio-temporal scales involved, as well as to its practical relevance to the flow of complex rheological fluids like biological fluids in a rotational frame, which are commonly used in medical diagnostics.

Here, we attempt to analyze the electrically driven transport of viscoelastic fluids in a rotating microfluidic channel. In addition to the rotational forces, we consider electro-osmotic effects due to interactions between an induced surface charge and a driving axial electric field. We consider thick but non-overlapping electrical double layers (EDLs) adhering to the channel walls, in order to resolve the charge distribution in the channel. Our results demonstrate that because of non-linearities in the flow, the resultant effect of rotational and electro-kinetically driven microfluidics is not a mere linear superposition. We believe that our results may be of potential interest towards designing rapid diagnostic devices in

* Corresponding author. Tel.: +913222282990.
E-mail address: suman@mech.iitkgp.ernet.in (S. Chakraborty).

http://dx.doi.org/10.1016/j.jnnfm.2016.03.006

List of Symbols

H	half-height of the channel, m
$\mathbf{E}$	applied electric field, V/m
$\mathbf{u}$	velocity vector, m/s
k_B	Boltzmann constant, m^2kg/s^2K
$n^{\pm}$	co-ion and counter-ion number density
n_{∞}	ionic concentration in the bulk
e	elementary electric charge, C
p	pressure, Pa
$\mathbf{T}$	fluid stress tensor, N/m^2
P	modified pressure, Pa
x	axial coordinate, m
y	transverse coordinate, m
z	transverse coordinate, m
u	velocity in x-direction, m/s
v	velocity in y-direction, m/s
w	velocity in z-direction, m/s
T_{xx}	fluid stress tensor component along x direction in y-z plane, N/m^2
T_{xy}	fluid stress tensor component along y direction in y-z plane, N/m^2
T_{xz}	fluid stress tensor component along z direction in y-z plane, N/m^2
T_{yx}	fluid stress tensor component along x direction in x-z plane, N/m^2
T_{yy}	fluid stress tensor component along y direction in x-z plane, N/m^2
T_{yz}	fluid stress tensor component along z direction in x-z plane, N/m^2
T_{zx}	fluid stress tensor component along x direction in x-y plane, N/m^2
T_{yz}	fluid stress tensor component along y direction in x-y plane, N/m^2
T_{zz}	fluid stress tensor component along z direction in x-y plane, N/m^2
u_{HS}	Helmholtz-Smoluchowski (HS) velocity, m/s
T	absolute temperature, K
t	time, s
Q	volumetric flow rate, $m^3/s/m$
Re_{Ω}	rotational Reynolds number, $\rho UH/\mu$
Wi	Weissenberg number

Greek symbols

$\boldsymbol{\Omega}$	angular velocity vector, rad/s
ρ	density of the fluid, kg/m^3
ρ_e	ionic charge density, C/m^3
ψ	electrical potential, V
λ	EDL thickness, nm
κ	Debye–Hückel parameter (inverse of EDL thickness)
η	dynamic viscosity, Pa-s
λ_1	fluid relaxation time, s
λ_2	fluid relaxation time, s
β	ratio of the relaxation to the retardation times of the fluid
ς	ion valence
ε	permittivity, C/Vm
θ	angle of flow

Subscripts

w	wall
x	along x-direction
y	along y-direction

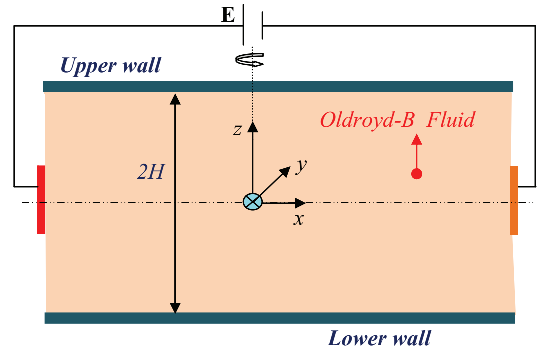

Fig. 1. (Color online) Schematic depicting the problem considered in the present study. The physical dimensions of the channel along with the direction of applied forces are shown in the schematic. The channel is rotating about z-axis with a constant angular velocity Ω. The coordinate system is also rotating with the channel. The applied electric field $\mathbf{E}$ makes the flow occur in x-direction (axial) of the channel, while the channel rotation induced the flow velocity in the lateral direction (y-direction) as well.

rotationally actuated environment, simultaneously tuned by electro-kinetic influences.

2. Problem description and mathematical formulation of the problem

We consider an unsteady flow of viscoelastic fluid in a rotating microfluidic channel as schematically depicted in Fig. 1. We here describe the flow dynamics with respect to the coordinate system, which is rotating with the channel itself (see Fig. 1). The coordinate axes x, y and z are directed along the length, width and height of the channel respectively. The channel is rotating about z axis at a constant angular velocity $\boldsymbol{\Omega} = (0,\ 0,\ \Omega)$. We consider that the length and width of the channel are much larger than its height, i.e., length>width>>height. The fluid is considered to be initially at rest, while the combined consequences of the applied electric field $\mathbf{E} = (E_x, 0, 0)$ in the x-direction along with the rotational effect of the channel along z-direction make the flow occur in the channel. In the present study, we consider Oldroyd-B model for representing the constitutive behavior of the viscoelastic fluid. We further consider the height of the channel to be $2H$.

2.1. The velocity distribution in the flow field

The governing equations of the flow dynamics, relative to the frame of the rotating channel as considered in this study, are given by [26–28],

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + 2\left(\vec{\Omega}\times\mathbf{u}\right) + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\right) = -\nabla p + \nabla\cdot\mathbf{T} + \rho_e\mathbf{E} \tag{2}$$

In Eq. (2), $\mathbf{T}$ is the stress tensor and $\rho_e\mathbf{E}$ is the electro-osmotic body force per unit volume, $\mathbf{u}$ is the velocity vector, $\mathbf{r}$ is the radial coordinate and $\boldsymbol{\Omega}$ is the angular velocity vector. The stress tensor equation following the Oldroyd-B constitutive model equation is given by [29,30],

$$\mathbf{T} + \lambda_1 \overset{\nabla}{\mathbf{T}} = 2\eta\left(\mathbf{D} + \lambda_2 \overset{\nabla}{\mathbf{D}}\right) \tag{3}$$

where, the upper convected derivative of $\mathbf{T}$ is defined as [31],

$$\overset{\nabla}{\mathbf{T}} = \frac{\partial \mathbf{T}}{\partial t} + (\mathbf{u}\cdot\nabla)\cdot\mathbf{T} - (\nabla\mathbf{u})\cdot\mathbf{T} - \mathbf{T}\cdot(\nabla\mathbf{u})^T \tag{4}$$