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Analytical modelling and numerical verification of non-Newtonian fluid flow through and over two-dimensional porous media

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ABSTRACT

In this paper, analytical equations are derived for predicting the flow of time independent non-Newtonian fluids through a two-dimensional granular porous structure and validated numerically. These equations were then used in the analytical expressions for the volumetric flow rates of generalised Newtonian fluids flowing inside a composite channel, consisting of a free-flow region adjacent to an infinite porous domain. The analytical equations are validated numerically and modifications are proposed. Commercial CFD software is used in these numerical validations where fluid flow is considered at a continuum level.

1. Introduction

Analytical models were proposed by Cloete and Smit [10] for Herschel–Bulkley fluids [5,25] traversing both three-dimensional consolidated (i.e. sponges or foams) and unconsolidated (i.e. granular media) porous regions. These porous domains were assumed to be homogeneous and the effects of exterior macroscopic boundaries were neglected. By making appropriate substitutions, these models included Newtonian, Bingham plastic and power law fluids, and, without stating it explicitly, also Casson fluids. In this paper a similar analytical technique is followed to formulate expressions for unidirectional fibre beds to be applied to two-dimensional case studies.

In a later analytical study by Cloete et al. [11], the influence the exterior macroscopic boundaries has on the profile of the phase average of the velocity was estimated by assuming a Brinkman-like effect [4]. The porous region was assumed to be homogeneous up to these external boundaries and the phase average of the velocity was assumed to be zero at the exterior walls. In the present paper, the first of these assumptions is investigated and an alteration to the existing model [11], where a change in porosity close to the macroscopic external boundary has been incorporated, is proposed. In the literature, adjusting the porosity close to external boundaries is not an unfamiliar approach. Cohen and Metzner [12] considered flow through a cylindrical column packed with spheres. The cross-sectional area of the column was divided into three domains where the porosity was denoted as a spatial varying functions in the outer two regions. In addition, to incorporate the friction due to the column walls, the work of Mehta and Hawley [27] was followed where the hydraulic radius was redefined by adding the ratio between the wetted surface of the wall and the bed volume to the denominator. Both the porosity functions and the newly defined hydraulic radius were then utilised at the wall regions in the capillary tube model and the volumetric flow rate was determined by integrating over the three respective regions. Nield [29] used a two domain approach, yielding similar results to this three domain approach [12]. Rao and Chhabra [32] substituted the redefined hydraulic radius (also applied to a capillary-tube porous model) into the power law model proposed by Kemblowski and Michniewicz [22] without altering the porosity. Here [32], by altering the hydraulic radius, the wall effect is introduced globally over the entire porous structure. This method varies slightly from that of Cohen and Metzner [12], where the modified hydraulic radius (along with the varying porosity) was only used close to the exterior wall. Other studies where external macroscopic wall effects were incorporated without the alteration of porosity were e.g. Comiti and Renaud [13] (applied to Newtonian fluids) and Sabiri and Comiti [33] (for power law fluids) who made a global adjustment to the definition of the dynamic specific surface area by including the surface area of the cylindrical container.

In the present paper free-flow over a porous domain is considered. Flow in such a composite domain has long been under investigation. The pioneer work of Beavers and Joseph [3], that served as a corner stone for many later studies, focused on the flow of

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Newtonian fluids over a porous surface. This study was both analytical and experimental. They assumed that the forced flow in the open channel was described by Stokes' equation and below a transition layer (where shear effects are transmitted into the porous structure), in the isotropic, homogeneous porous medium, Darcy's law was assumed to be abided by. The velocity profile in the transition layer was not modelled analytically. However, from experimental results they observed that the velocity at the porous-fluid interface can be much greater than the Darcy velocity. Semi-empirical expressions for the flow in the free-flow domain and velocity at the porous-fluid interface were derived. In a later study by Neale and Nader [28] the fully developed velocity profiles of both regions were derived analytically for Newtonian fluids. The same modelling procedure as Beavers and Joseph [3] was followed for flow in the free-flow region and the profile of the averaged velocity in the porous medium was obtained by solving Brinkman's equation [4]. Shear stress continuity was enforced as a boundary condition at the fluid-porous interface. Cloete et al. [11] followed the technique of Neale and Nader [28] and applied it to non-Newtonian fluids and implemented earlier derived apparent permeability models [10].

In the present study a model estimating the permeability of an infinite two-dimensional porous medium is derived. These permeabities were then implemented in the model of Cloete et al. [11] for flow over such a two-dimensional porous structure. The results from these analytical expressions are validated with numerically obtained data and modifications are proposed.

1.1. Some background on flow through porous media: Volume averaged momentum transport equation

By making use of standard volume averaging techniques [1], the momentum transport equation and the continuity equation, governing flow in a porous medium at a continuum level, may be written into macroscopic form. It is assumed that the porous structure is rigid, the fluid is incompressible and homogeneous and that the no-slip boundary condition is applicable at the interstitial fluid-solid interfaces. The representative elementary volume (REV) is defined as an arbitrary volume which is statistically representative of all the physical properties, of the porous domain, in the immediate vicinity of the point where the centroid of the REV is located. Volume averaging over an REV with volume $\mathcal{V}_o$ yields the following equation governing the average fluid flow through any type of porous structure [11]:

$$\rho \frac{\partial \mathbf{q}}{\partial t} + \rho\, \nabla\cdot[\mathbf{uq}] = -\varepsilon \nabla \langle p \rangle_f + \nabla\cdot[\langle \eta \rangle_f \nabla \mathbf{q}] + \nabla \mathbf{q} \cdot \nabla \langle \eta \rangle_f - \frac{1}{\mathcal{V}_o} \iint_{\mathcal{S}_{fs}} (\{p\}\mathbf{n} - \mathbf{n}\cdot \eta \nabla \mathbf{v}) d\mathcal{S}. \quad (1)$$

In this paper, $\langle\ \rangle_f$ and $\{\}$ denote the intrinsic phase average and the deviation at a point with respect to the intrinsic phase average, respectively, for any given quantity defined inside the fluid phase region within an REV. As customary, ρ, p and η denote the fluid density, the static pressure and the apparent viscosity of the non-Newtonian fluid, respectively. The total fluid-solid interface inside the REV is denoted by $\mathcal{S}_{fs}$ and the local porosity (void space fraction) of the considered REV is represented by ε. In Eq. (1), $\mathbf{u}$ and $\mathbf{q}$ denote the intrinsic phase average and the phase average of the velocity, respectively. Both these velocity averages are orientated in the local streamwise direction, $\hat{\mathbf{n}}$, and following from the Dupuit–Forchheimer relationship [17], $\mathbf{q} = \varepsilon \mathbf{u}$. The velocity at a continuum level inside the interstitial pores is represented by $\mathbf{v}$. The unit vector, $\mathbf{n}$, is the normal vector on $\mathcal{S}_{fs}$ as well as the fluid-fluid interfaces on the outer surface of the REV and is directed into the solid phase or to the outside of the REV. In the derivation process of Eq. (1), following Bear and Bachmat [2] where the momentum dispersion term was assumed to be negligibly small in comparison to the macroscopic convection term, Cloete et al. [11] made a similar assumption regarding the volume averaged diffusion term for generalised Newtonian fluids.

The only applicable body force is gravity and therefore all three vector components of the body force are uniform vector fields. Since the curl of both a uniform vector field and the gradient of a quantity is zero, the body force may be expressed as a gradient and incorporated as part of the pressure gradient term. Thus, in Eq. (1) and for the remainder of this paper, $\nabla \langle p \rangle_f$ incorporates both the static pressure gradient as well as the body force.

2. Infinite porous domain

If the effect due to external boundaries is disregarded and the phase average of the velocity is assumed to be time independent and uniform, Eq. (1) reduces to

$$-\nabla \langle p \rangle_f = \frac{1}{\mathcal{V}_f} \iint_{\mathcal{S}_{fs}} (\{p\}\mathbf{n} - \mathbf{n}\cdot \eta \nabla \mathbf{v}) d\mathcal{S}, \quad (2)$$

where $\mathcal{V}_f$ denotes the volume occupied by fluid within the REV, i.e. $\mathcal{V}_f = \varepsilon \mathcal{V}_o$. In order to solve this integral, the interstitial velocity gradients at the fluid-solid interfaces of the pores (of which the geometry must also be known) is required. Therefore pore-scale models have to be utilised to estimate the resisting force exerted by the porous structure on the traversing fluid.

2.1. Secondary averaging

By making use of modified representative unit cell (RUC) models (an idea that was initiated by Du Plessis and Masliyah [15]), Cloete and Smit [10] proposed analytical expressions to estimate the magnitude of the integral in Eq. (2) for Herschel–Bulkley fluids traversing both consolidated and unconsolidated porous media. In the RUC modelling technique only the average morphology over an REV is required. This model is based on the assumption that the fluid traverses the porous medium in imaginary streamtubes and, in a three-dimensional case study, four of the outer surfaces of the RUC are orientated parallel and two faces are orientated perpendicular to an enclosed streamtube. An RUC is defined as the smallest possible cell in which the statistical average geometrical properties of the REV it is representing (e.g. the local average porosity and the tortuosity) and the local flow conditions (e.g. the gradient of the intrinsic phase average of the pressure) can be imbedded. The centroid of the REV and corresponding RUC coincides and the REVs and RUCs pertaining to two adjacent mathematical points thus overlap. The RUC should therefore not be viewed as a repetitive building block with which the porous medium can be reconstructed theoretically, as is the case in some other unit cell models [19].

Eq. (2) may therefore be rewritten in terms of RUC-dimensions as follows

$$-\nabla \langle p \rangle_f = \frac{1}{V_f} \iint_{S_{fs}} (\{p\}\mathbf{n} - \mathbf{n}\cdot \eta \nabla \mathbf{v}) dS, \quad (3)$$

where V_f and S_{fs} represent the volume occupied by the fluid phase and the total fluid-solid interfaces, now with respect to an RUC.

The core of the RUC-model (as it was adapted by Du Plessis and Diedericks [14]) was based on finding an expression for the interstitial wall shear stress, $|\tau_{w\|}|$, in terms of the average fluid speed within the local streamwise orientated interstitial channels, $\bar{v}_{\|}$. For viscoplastic fluids, this was accomplished by Cloete and Smit [10]by making use of the asymptote matching technique that was proposed by Churchill and Usagi [7]. Here, the two limiting conditions considered for the wall shear stress were where no-shearing effects occur and a no-plug power law limit has been reached: