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Peristaltic flow of Bingham fluids at large Reynolds numbers: A numerical study

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ABSTRACT

Peristaltic flow of a viscoplastic fluid which obeys the bi-viscous (Bingham) model as its constitutive equation is numerically studied in a planar two-dimensional channel using the multiple-relaxation-time lattice Boltzmann method (MRT-LBM). Numerical results could be obtained at large Reynolds numbers for arbitrary set of wavelength and amplitude ratios of the peristaltic wave. It is shown that depending on the Reynolds number, a fluid's yield stress may increase or decrease the time-mean flow rate of peristaltic pumps. Our numerical results also show that for yield-stress fluids there exists a threshold wave number above which the yield stress can accelerate fluid transport whereas below which it can have a decelerating effect. The yield stress is also predicted to diminish the size of the fluid bolus in the "trap" phenomenon for certain set of parameters.

1. Introduction

A wave propagating along the flexible wall(s) of a channel is known to force the contained fluid to flow in the same direction, even when there is no external pressure gradient involved. The flow, which is referred to as *peristalsis,* is of prime importance in biomechanics as it is the mechanism through which physiological fluids such as blood and urine are transported in human body. Further interest in this type of flow originates from the fact that it is well-suited for the transport of highly-viscous fluids such as high solids slurries. In addition, since cross-contamination with exposed pump components cannot occur in this type of flow, it has found widespread application in pump industry for the transport of corrosive and sterile fluids. For reasons like these, peristalsis has been the subject of intensive studies in the past, in both theoretical and experimental domains alike [1–10]. In the theoretical domain, simplified models have been developed in circular and planar channels to better understand peristalsis and the parameters influencing its performance as a means of fluid transport. For ease of analysis, early studies in this area have relied on several simplifying assumptions chief among them are: (i) the Reynolds number is vanishingly small, (ii) the wavelength is infinitely long, and (iii) the wave amplitude is sufficiently small [6,7]. The creeping-flow assumption has been an essential part of these studies as it allows the governing equations to be reduced to that of the Stokes flow. The long wavelength assumption, either separately or in conjunction with the small amplitude-ratio assumption, has also proven very useful as it allows perturbation techniques to be invoked for obtaining an analytical or semi-analytical solution. None of these assumptions are, strictly speaking, valid in the pump industry. Still, these approximate studies have been quite successful in identifying the key role played by a fluid's viscosity on subtle issues such as the *trap* and *reflux* phenomena as experimentally observed in peristaltic flows [1–5]. Quantitatively, however, comparison with experimental results is not always so great [8–10].

The discrepancy between theory and experiment is often realized to have arisen from a violation of, at least, one or more of the above-mentioned assumptions [11–13]. Another source of discrepancy can be traced back to the notion that the test fluid used in these experimental studies was more or less non-Newtonian. Physiological fluids such as blood and industrial fluids such as drilling mud are known to exhibit a variety of non-Newtonian behaviors. And, it is well established that a fluid's rheology can affect peristaltic flows in a non-trivial way. For example, Boheme and Friedrich [14], and Siddiqui and Schwarz [15] have shown that a fluid's elasticity lowers the flow rate in peristaltic flows, see also Refs. [16–21]. A similar result has been found by Vajravelu et al. [22] as to the effect of a fluid's yield stress on such flows, see also Ref. [23]. Shear-thinning, on the other hand, has been shown by Rao and Mishra [24] to increase the flow rate of peristaltic flows.

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Nomenclature

a	half-height of the unperturbed channel
b	amplitude of the travelling wave
λ	wavelength
c	wave speed
x	longitudinal coordinate in the laboratory frame
y	longitudinal coordinate in the laboratory frame
h	height of a point on the wall in the laboratory frame
t	time
h_u	lateral coordinate of the upper wall
h_l	lateral coordinate of the lower wall
h_{max}	half of the maximum height of the channel
v_u	velocity of the upper wall
v_l	velocity of the lower wall
$\mathbf{u}$	velocity vector of the fluid
ρ	fluid's density
p	isotropic pressure
$\underset{\sim}{\tau}$	extra stress tensor
d_{ij}	rate-of-deformation tensor
τ_y	fluid's yield stress
μ_0	viscosity of the un-yielded region in bi-viscous Bingham model
μ_p	viscosity of the yielded region
$\dot{\gamma}$	second invariant of rate-of-deformation tensor
$\dot{\gamma}_c$	critical shear rate of the bi-viscous Bingham model
$\mathbf{c}$	velocity vectors in lattice Boltzmann method
f	probability distribution function
$\mathbf{\Omega}$	collision matrix
f_i^{eq}	equilibrium distribution function
ξ	relaxation time in the SRT-LBM
$\mathbf{I}$	identity matrix
$\mathbf{M}$	transformation matrix in the MRT-LBM
$\mathbf{S}$	relaxation matrix in the MRT-LBM
Δ	ratio of the wetted fluid–solid link to its total length
α	wave number
ϕ	amplitude ratio
Bn	Bingham number
Re	Reynolds number
T	period of the travelling wave
q	time-mean flow rate in the laboratory frame of reference
Q	dimensionless time mean flow rate in the laboratory frame of reference

Although the above works have shed some light onto our understanding of the role played by a fluid's rheology on the peristaltic flow, an extension of these predictions to the industrial situations is not straightforward. This is because these theoretical results have been obtained based on restrictive assumptions quite similar to those mentioned above for Newtonian fluids. These assumptions, which are questionable even for Newtonian fluids, are less plausible for non-Newtonian fluids, due mainly to the nonlinearity of their constitutive behavior. This notion has beautifully been demonstrated by Ceniceros and Fisher [25] when dealing with the peristaltic flow of a viscoelastic fluid obeying the Oldroyd-B model. They have numerically shown that, contrary to the published data obtained using the lubrication theory [20], a fluid's elasticity may increase or decrease the flow rate of peristaltic pumps depending on the Weissenberg number of the flow and the amplitude ratio of the propagating wave. In the present work, we show that this is also true when dealing with peristaltic flow of viscoplastic fluids. That is, it will be shown that depending on the Reynolds number and the wavelength of the peristaltic wave, a fluid's yield stress may increase or decrease the flow rate of peristaltic pumps. To show this, we rely on a full numerical analysis based on the lattice Boltzmann method [26]. This robust numerical method has recently been used with great success for simulating peristaltic transport of a two-dimensional particle immersed in a Newtonian liquid at moderate Reynolds numbers [27].

To reach its objectives, the work is organized as follows. We start with developing the mathematical framework for peristaltic transport of a viscoplastic fluid obeying the bi-viscous (Bingham) model in a planar two-dimensional channel. We then proceed with describing the numerical method of solution (i.e., the lattice Boltzmann method) in some details. Numerical results are presented next addressing the effect of the yield stress on the flow characteristics for a wide range of Reynolds numbers, wave numbers, and amplitude ratios–to the best of our knowledge, for the first time.

2. Mathematical formulation

We consider peristaltic flow of a viscoplastic fluid in a planar channel under laminar and two-dimensional conditions. Our interest in this particular geometry arises from the fact that it provides a greater flexibility over its tubular version in peristalsis studies. That is, we can fix one of the walls and investigate the asymmetric case, or move the walls with a phase lag to see how it affects the flow characteristics. In addition, it more easily yields itself to an experimental investigation when needed. In the present work, however, we are interested in the symmetric case only, as shown in Fig. 1.

To be able to verify our code with published data, we assume that the waves propagating along the upper and lower walls of the channel are of the following forms [1–5]:

$$h_u(x,t) = +a - b\cos\left[\frac{2\pi}{\lambda}(x-ct)\right] \tag{1}$$

$$h_l(x,t) = -a + b\cos\left[\frac{2\pi}{\lambda}(x-ct)\right] \tag{2}$$

where subscripts "u" and "l" refer to the upper and lower walls, respectively, a is the half-height of the unperturbed channel, b is the amplitude of the travelling wave, λ is the wavelength, and c is the wave speed (see Fig. 1). With the assumption that the channel is very long so that an infinite (integral) number of waves are traveling along the upper and lower walls, we allow ourselves to focus on just one wave front, far from the two ends of the channel. This also means that the pressure on both ends of the domain is the same for each wave while there might be a pressure gradient within the domain itself. To simplify the analysis, we ignore the effects of the gravitational force as compared with the other forces involved in the channel. In addition, any coupling between the fluid and the elastic behavior of the wall material, which is assumed to be mass-less, is ignored. We assume that initially the fluid in the whole domain is at rest. At time

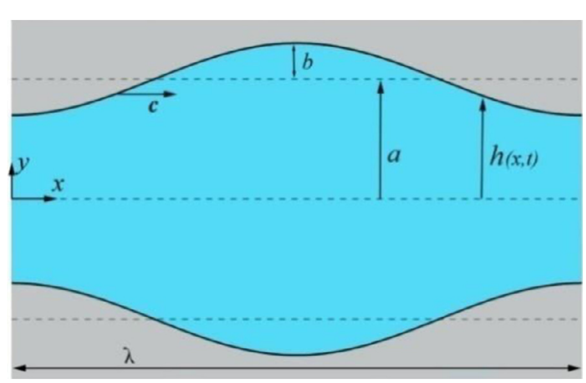

Fig. 1. Schematic showing the flow domain and its geometrical parameters.