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Reynolds-averaged modeling of turbulent flows of power-law fluids

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ABSTRACT

The paper presents a novel Reynolds-averaged turbulence model for flows of power-law fluid. The model uses the elliptic relaxation approach to capture the near-wall turbulence anisotropy. The turbulence model for Newtonians fluids is modified by introducing closed approximations of correlations between velocity and viscosity fluctuations. The approximation for non-Newtonian extra stress is derived with the assumption of smallness of molecular viscosity fluctuations. A closed model for the averaged molecular viscosity is derived which takes into account its nonlinear dependence on the shear rate. Validation of the model against the direct numerical simulation (DNS) data for power-law fluids flows in the pipe demonstrates that new model is able to predict the main features of the non-Newtonian turbulence. Mean velocity, turbulent energy and averaged molecular viscosity distributions agree well with DNS data.

1. Introduction

Studies of non-Newtonian fluids have been motivated by numerous applications since the dependence of the stress tensor on the strain rate tensor for many important classes of fluids (polymeric fluids, drilling fluids, suspensions, etc.) is nonlinear. Non-Newtonian fluid flows in the channels of many industrial settings (different heat exchangers, bearings, centrifuges, drill string, etc.) are typically turbulent. Experimental results show that non-Newtonian rheology has effect on the turbulent structure of the flow. The turbulence intensities normal to the wall are lower for non-Newtonian fluids than for Newtonian ones, while the streamwise turbulent fluctuations may be even higher. Such modification of near-wall turbulence leads to friction drag reduction with respect to the Newtonian flow. Another experimental observation is that transition to turbulence may be delayed in non-Newtonian fluids.

The most widely used non-Newtonian fluids in engineering applications have shear-thinning behavior. Shear-thinning fluids are characterized by apparent viscosity which gradually decreases with increasing shear rate. An important class of shear-thinning fluids is so-called power-law fluids, whose viscosity decreases with increasing rate of fluid deformation according to the power law. In constructing a turbulence model for such fluid, it is necessary to solve two problems: (1) modeling of the averaged molecular viscosity and (2) modeling of correlations not typical of Newtonian fluids.

A few studies have focused on the development of a mathematical model and its corresponding numerical algorithm for turbulent flows of shear-thinning fluids. One of the first modeling studies of turbulent flows of power-law fluids was performed by Malin [1], who used the low Reynolds number k-ε model of Lam–Bremhorst [2]. In order to take into account the influence of power-law rheology on the flow turbulent structure Malin proposed a modification of a near-wall damping function for eddy viscosity. The modified damping function depended on the rheology parameters (power index). The introduced model factor was adapted to the experimental correlation [3] for friction factor depending on the Reynolds number and the power index. Mean velocity profiles obtained by the developed model were in good agreement with available experimental data. Except for the empirically modified damping function Malin did not include any effects of non-Newtonian viscosity on turbulence.

A more recent model is presented in the work of Pinho [4], where a model is formulated for the averaged molecular viscosity and an estimate is made of the non-Newtonian correlation terms appearing in the k-ε model. Then, Cruz and Pinho [5] have developed a turbulence model for a version of a generalized Newtonian fluid. The turbulence model was based on the Nagano and Hishida k–ε model and used to simulate the turbulent pipe flow of various polymer solutions. In the next work Cruz et al. [6] improved the turbulence model by introducing a closed model for non-Newtonian stress terms in momentum equation and its corresponding effect on the transport of turbulent kinetic energy. For the pipe flow of shear-thinning fluids the developed model predicts well the friction factor and mean velocity. However, the comparison with direct numerical simulation (DNS) results has demonstrated the need for the turbulence model

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modifications which can describe properly the anisotropic behavior of non-Newtonian turbulence.

Flows of shear-thinning fluids are characterized by greater anisotropy of turbulent fluctuations than Newtonian shear flows, i.e. by the dominance of streamwise fluctuations over the fluctuations normal to the wall. For this reason, the turbulence model using damping functions to describe the wall effect should be modified, and this was done in [1]. The main drawback of this approach is that nonlocal effects are described by functions that depend on local values. Models with local damping functions usually do not differentiate between the viscous and nonviscous damping effects of the wall. An increase in molecular viscosity away from the wall may be perceived as a signal of the proximity of the wall and lead to activation of the damping function.

In constructing a turbulence model for viscoelastic media, Iaccarino et al. [7] use Durbin's approach [8], in which the nonviscous damping effect of the wall is described using a turbulent energy redistribution mechanism. Masoudian et al. [9] using the procedure suggested by Iaccarino et al. [7] proposed a new k–ε–$\upsilon 2$–f model for modeling turbulent channel flow of dilute polymer solutions. The proposed model includes new closures for the nonlinear fluctuating terms appearing in the rheological constitutive equation and for the polymer stress work term. The results demonstrated that the developed turbulence model does not require any wall damping functions and has a good predictive capability.

As postulated in the k–ε–$\upsilon 2$–f model for Newtonian fluids [8], the eddy viscosity near the wall is defined by wall-normal velocity fluctuations. Although this approach is phenomenological, it does not include any local correction functions that take into account the proximity of the wall. Therefore, it is natural to generalize this approach to the case of power-law fluids. This is the aim of this work.

A model for the turbulent flows of power-law fluids is presented. A turbulence model for Newtonian fluids based on Reynolds-Averaged Navier–Stokes equations is modified by introducing closed approximations for non-Newtonian stress and for correlation between fluctuating velocity and viscosity. The averaged molecular viscosity is modeled in the same way as in [10]. The modeling results are compared with the data obtained by direct numerical simulation (DNS) of Rudman et al. [11, 12] covering four different power law indices. Also the behavior of the turbulence model is assessed by comparing numerical results with the corresponding experimental data for the friction factor [3].

2. Reynolds averaging of transport equations

We consider nonisothermal incompressible power-law fluid flow. Transport equations for the instantaneous variables (hereinafter they are marked with a hat) of this fluid have the form

$$\nabla\cdot\hat{\mathbf{u}} = 0,\quad \rho\frac{\partial\hat{\mathbf{u}}}{\partial t} + \rho\hat{\mathbf{u}}\cdot\nabla\hat{\mathbf{u}} = -\nabla\hat{p} + \nabla\hat{\tau}, \tag{1.1}$$

where ρ is the density of the fluid, $\hat{\mathbf{u}}$ is its velocity, and $\hat{p}$ is the pressure. For the power-law fluids considered, the stress tensor $\hat{\tau}$ is proportional to the strain rate tensor $\hat{\mathbf{S}}$

$$\hat{\tau} = 2\hat{\mu}\hat{\mathbf{S}},\quad \hat{\mathbf{S}} \equiv \hat{S}_{ij} = \frac{1}{2}\left(\frac{\partial\hat{u}_i}{\partial x_j} + \frac{\partial\hat{u}_j}{\partial x_i}\right), \tag{1.2}$$

and the apparent viscosity $\hat{\mu}$ depends on the second invariant of the strain rate tensor $\hat{\dot{\gamma}} = 2\hat{S}_{ij}\hat{S}_{ij} = \hat{S}^2$ as follows:

$$\hat{\mu} = k_v\hat{\dot{\gamma}}^{\,n-1}. \tag{1.3}$$

Here n is the flow index and k_v is the consistency factor. In the rest of the paper, we will use the second invariant of the strain rate tensor $\dot{\gamma}$ as shear rate.

In the following, turbulent flows are described using Reynolds averaging approach. The instantaneous velocity field is then represented as the sum of the average, **U,** and fluctuating, **u′** velocity fields: $\hat{\mathbf{u}} = \mathbf{U} + \mathbf{u}'$. The other variables are represented similarly. In view of the above, averaging equation (1.1) we obtain

$$\nabla\cdot\mathbf{U} = 0,\quad \rho\mathbf{U}\cdot\nabla\mathbf{U} = -\nabla p + \nabla\cdot(2\mu\mathbf{S}) - \rho\nabla\cdot\left\langle\mathbf{u}'\mathbf{u}'\right\rangle + \nabla\cdot\tau_N, \tag{1.4}$$

where **S** is the averaged strain rate tensor, μ is the averaged viscosity, p is the averaged pressure, and $-\langle\mathbf{u}'\mathbf{u}'\rangle$ is the symmetric Reynolds stress tensor. Here and below, the angular brackets denote Reynolds averaging. It should be borne in mind that due to the nonlinear rheological dependence (1.3), Eq. (1.4) contain additional stress associated with the dependence of the viscosity on the strain rate of the fluid

$$\tau_N = 2\left\langle\mu'\mathbf{S}'\right\rangle, \tag{1.5}$$

which is hereinafter referred to as the non-Newtonian stress tensor.

To close Eq. (1.4), it is necessary to have expressions for the last two terms in the equation of motion and an expression defining the averaged viscosity. The Reynolds stress transport equation is obtained in the usual way from the equations for the averaged velocities and Eq. (1.1) and has the form

$$\rho U_k\frac{\partial}{\partial x_k}\left\langle u_i'u'_j\right\rangle = \rho\left(P_{ij} + D_{t,ij} + D_{\nu,ij} - \varepsilon_{ij} + \Phi_{ij}\right) + \Gamma_{N,ij} + D_{N,ij}, \tag{1.6}$$

where

$$P_{ij} = -\left(\left\langle u_i'u'_k\right\rangle\frac{\partial U_j}{\partial x_k} + \left\langle u'_ju'_k\right\rangle\frac{\partial U_i}{\partial x_k}\right), \tag{1.7}$$

$$\Phi_{ij} = \left\langle\frac{p'}{\rho}\left(\frac{\partial u'_j}{\partial x_i} + \frac{\partial u'_i}{\partial x_j}\right)\right\rangle, \tag{1.8}$$

$$D_{t,ij} = -\frac{\partial}{\partial x_k}\left(\left\langle u'_iu'_ju'_k\right\rangle + \left\langle\frac{p'}{\rho}\left(\delta_{jk}u'_i + \delta_{ik}u'_j\right)\right\rangle\right), \tag{1.9}$$

$$D_{\nu,ij} = \frac{\partial}{\partial x_k}\left(\frac{\mu}{\rho}\frac{\partial\left\langle u'_iu'_j\right\rangle}{\partial x_k}\right), \tag{1.10}$$

$$\varepsilon_{ij} = 2\frac{\mu}{\rho}\left\langle\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_j}{\partial x_k}\right\rangle. \tag{1.11}$$

Here P_{ij} on the right side of Eq. (1.6) is the stress production tensor; $D_{t,ij,}$and D_{vij} are the diffusive transport due to turbulent fluctuations and molecular transport, respectively; Φ_{ij} is the pressure–strain correlation or redistribution term; ε_{ij} is the viscous dissipation rate of turbulent stresses.

The last two terms in Eq. (1.6) are due to viscosity fluctuations. The first of these

$$\Gamma_{N,ij} = -\left\langle\mu'\frac{\partial u'_i}{\partial x_k}2\hat{S}_{kj}\right\rangle - \left\langle\mu'\frac{\partial u'_j}{\partial x_k}2\hat{S}_{ki}\right\rangle \tag{1.12}$$

is the work of non-Newtonian stresses, and the second

$$D_{N,ij} = \frac{\partial}{\partial x_k}\left[\left\langle\mu'u'_i2\hat{S}_{kj}\right\rangle + \left\langle\mu'u'_j2\hat{S}_{ki}\right\rangle\right] \tag{1.13}$$

is the diffusion due to viscosity fluctuations.

3. Turbulence model

We use the ζ–f model of Hanjalić et al. [13], which is a version of the Durbin model [8], as the baseline turbulence model. Its advantage is that it accurately takes into account the anisotropy of the near-wall turbulence. The model does not use any wall damping function that