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Short Communication

Flow of truncated power-law fluid between parallel walls for hydraulic fracturing applications

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ABSTRACT

An essential closure of hydraulic fracturing models is the solution of the momentum equation for flow between plane parallel walls. Newtonian or simple power-law rheology is usually assumed. In real treatments, fracturing fluid often has more complicated rheology, such as Carreau. An earlier introduced modification to the power-law model enables a fair approximation to Carreau rheology. Unlike Carreau, it also enables a closed-form solution for the flow rate between plane parallel walls. The computational cost is, however, considerably smaller than with Carreau. Closed-form solution for the flow rate *versus* pressure gradient is obtained which is useful in hydraulic fracturing simulations. Compared to simple power-law model, the truncated power-law model improves accuracy of flow computations in small-aperture and large-aperture parts of the fracture, thereby improving the overall accuracy of hydraulic fracturing simulation.

1. Introduction

Hydraulic fracturing is one of the most popular well stimulation techniques in oil and gas industry. A fracture is created by injecting fluids or fluid–solids mixtures from a well into the surrounding rock. The permeability of the resulting fracture is typically orders of magnitude higher than that of the rock, which means that the fracture can act as a high-conductivity conduit from the reservoir towards the producing well. The success of the stimulation job depends on our ability to predict fracture growth and its final size by numerical modelling when designing the treatment [1]. Such modelling is only possible if the fluid flow in the growing fracture is predicted correctly.

Typically, non-Newtonian power-law fluids are used for hydraulic fracturing in conventional oil and gas reservoirs [2–6]. Fluid flow in the fracture at low Reynolds numbers is usually computed using the so-called lubrication theory approximation, whereby it is assumed that the fracture roughness is relatively small so that the flow can locally be approximated as flow between plane parallel walls [7–9]. The analytical solution of the momentum equation for the latter is then fed into the mass conservation equation for flow in the fracture, thereby eliminating one spatial dimension from the problem, i.e. the dimension along the fracture aperture. It should be noted, however, that this approach is computationally efficient only if a closed-form solution for flow between plane parallel walls is available. Therefore, the pure power-law model (Ostwald – de Waele model) has been routinely used in hydraulic fracturing simulators [3,10], with the apparent dynamic viscosity given by:

$$\eta_a = C\dot{\gamma}^{n-1} \quad (1)$$

In Eq. (1) C is the consistency index; n is the flow behaviour index; $\dot{\gamma}$ is the shear rate, $\dot{\gamma} = \sqrt{2\mathbf{D} : \mathbf{D}}$, where $\mathbf{D}$ is the strain rate tensor.

It has been pointed out by Shah and Yortsos [11] that the simple, two-parameter pure power-law model represented by Eq. (1) cannot correctly describe the behaviour of real fluids at low and high shear rates. In particular, for shear-thinning fluids, Eq. (1) implies that the apparent viscosity increases towards infinity as $\dot{\gamma} \to 0$, and decreases towards zero as $\dot{\gamma} \to \infty$. The importance of including both low shear rate and high shear rate cutoff viscosities into the rheological model of fracturing fluids has been emphasized e.g. by Oeth et al. [12]. Four-parameter models, such as Carreau or Cross models, represent more accurate descriptions of power-law fluid behaviour at high and low shear rates by introducing asymptotic values of the apparent viscosity at those limits. However, these models do not allow a closed-form solution for the flow between parallel walls. As a result, obtaining the flow rate as a function of the pressure gradient with these models would require a numerical integration along the conduit aperture in each grid point on the fracture surface. Such quadrature would require

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at least 5–6 integration points along half-aperture, in order to ensure sufficient accuracy. This is illustrated by a numerical example in Fig. 1 where the flow rate obtained by trapezoidal integration of the velocity profile is shown as a function of the number of integration points along the aperture. For Carreau or Cross rheologies, a nonlinear algebraic equation needs to be solved at each of these integration points by e.g. Newton's method to obtain the shear rate. This involves evaluation of power functions at each step of Newton's iteration. The use of the quadrature entails therefore a significant computational cost. For this reason, notwithstanding their improved accuracy, Carreau and Cross models have not been used in routine hydraulic fracturing simulations.

An improvement on the currently widely used pure power-law fluid model can be achieved by a well-known modification, namely by using cut-off viscosities at low and high shear rates (Section 2). Even though such modification is trivial, the behaviour of such fluid can be made to resemble Carreau or Cross fluids fairly well while the problem of flow between plane parallel walls remains analytically tractable. The closed-form solution for flow between plane parallel walls of such fluid is obtained in Section 3. The closed-form expression for the flow rate as a function of pressure gradient includes only one extra term. The computational overhead while using this model in hydraulic fracturing simulation is expected to be well justified by the improved accuracy of modelling the fluid behaviour.

2. Truncated power-law fluid

As mentioned in Section 1, an improvement on the pure power-law rheology can be achieved by using a four-parameter model, e.g. the Carreau fluid with the apparent viscosity represented by [13,14]:

$$\eta_a = \eta_\infty + (\eta_0 - \eta_\infty)\left(1 + (\lambda\dot{\gamma})^2\right)^{\frac{n-1}{2}} \tag{2}$$

where η_0 is the viscosity at zero shear rate; η_∞ is the limiting viscosity as $\dot{\gamma} \to \infty$; n and λ are fitting parameters. An example of the apparent viscosity *vs* shear rate curve in log–log coordinates for a shear-thinning Carreau fluid is given in Fig. 2 (dotted line), with the values of η_0, η_∞, n and λ given in Table 1.

The Carreau model describes many real fluids quite well, but cannot readily be used in a fracture flow simulation since it is not possible to obtain a closed-form solution for the Poiseuille-type flow between plane parallel walls with this model. A simple regularization of the pure power-law model described in [15] allows capturing basic features of the Carreau model, while at the same time enabling analytical treatment of the plane parallel wall flow problem, with a closed-form solution at the end of the road.

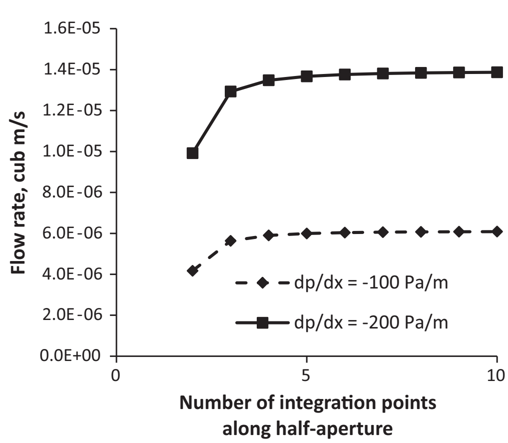

Fig. 1. Flow rate in a plane-wall conduit estimated by quadrature as a function of the number of integration points along half-aperture for Carreau fluid. Conduit aperture w = 1 mm. Conduit length in the direction perpendicular to flow is 1 m. Parameters of the Carreau fluid: low shear rate viscosity 0.5 Pa s; high shear rate viscosity 0.001 Pa s; exponent 0.25 (shear thinning); λ = 600 s [see Eq. (2) for a definition of Carreau fluid properties]. Two curves correspond to two values of the pressure gradient applied in the direction of flow.

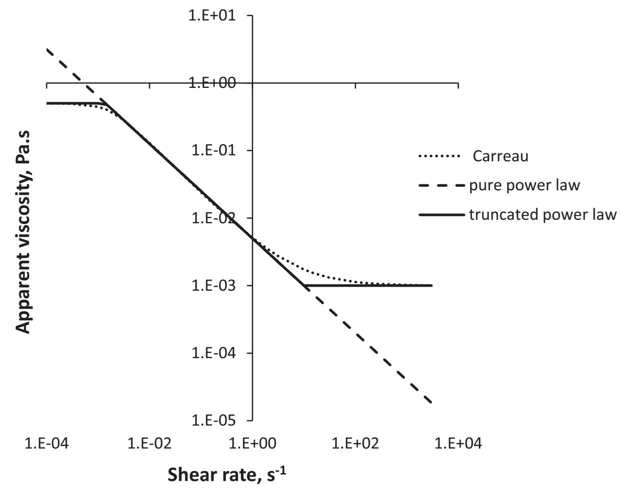

Fig. 2. Apparent viscosity *vs* shear rate for three rheological models. The values of model parameters are listed in Table 1.

Table 1
Parameters of the shear stress *vs* shear rate curves plotted in Fig. 2.

Rheological model	Parameter	Value
Carreau (dotted line in Fig. 2)	η_0, Pa s	0.5
	η_∞, Pa s	0.001
	n	0.25
	λ, s	600.0
Truncated power-law (solid line in Fig. 2)	η_0, Pa s	0.5
	η_∞, Pa s	0.001
	n	0.3
	C, Pa s^{2-n}	0.005
Pure power-law (dashed line in Fig. 2)	n	0.3
	C, Pa s^{2-n}	0.005

The truncated power-law model is constructed as follows [13]:

$$\eta_a = \begin{cases} \eta_0 & \text{for } \dot{\gamma} < \dot{\gamma}_1 \\ C\dot{\gamma}^{n-1} & \text{for } \dot{\gamma}_1 < \dot{\gamma} < \dot{\gamma}_2 \\ \eta_\infty & \text{for } \dot{\gamma} > \dot{\gamma}_2 \end{cases} \tag{3}$$

where $\dot{\gamma}_1 = (C/\eta_0)^{1/(1-n)}$ is the shear rate at which the low-viscosity cut-off is introduced; $\dot{\gamma}_2 = (C/\eta_\infty)^{1/(1-n)}$ is the shear rate at which the high-viscosity cut-off is introduced. The model represented by Eq. (3) has four parameters, similar to the Carreau model.

The truncated power-law model is illustrated in Fig. 2 (solid line), with the model parameters specified in Table 1. It is identical to a pure power-law model within the interval $\dot{\gamma}_1 < \dot{\gamma} < \dot{\gamma}_2$. The pure, two-parameter power-law model is also shown in Fig. 2 (dashed line).

3. Laminar flow between plane parallel walls

Our objective is to derive the solution to the problem of flow between parallel walls for the truncated power-law fluid described in Section 2. The geometry of the problem is shown in Fig. 3. The x-axis is directed along the centre line of the conduit, in the