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Stick-slip singularity of the Giesekus fluid

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ABSTRACT

The local asymptotic behaviour at the stick-slip singularity is determined for the Giesekus fluid in the presence of a solvent viscosity. In planar steady flow, the method of matched asymptotic expansions is used to show that it comprises a three region structure. Specifically, an outer or core region that links boundary layers at the rigid stick and free slip surfaces. In the outer region, the velocity field is shown to be Newtonian at leading order, with solvent stresses dominating the polymer stresses. In terms of the radial distance r from the singularity at the join of the stick and slip surfaces, the velocity field vanishes as $O(r^{\frac{1}{2}})$. Consequently, the singular velocity gradients and solvent stresses are of $O(r^{-\frac{1}{2}})$ with the less singular polymer stresses being shown to be $O(r^{-\frac{5}{16}})$. The solvent and polymer stresses become comparable near the rigid stick and free slip surfaces, where boundary layers are required. These are of thickness $O(r^{\frac{5}{4}})$ at the rigid stick surface and thickness $O(r^{\frac{17}{14}})$ at the free slip surface. Solutions are constructed for both stick-slip and slip-stick flow regimes. These asymptotic results do not hold for the Oldroyd-B model nor for the case when the solvent viscosity is absent.

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1. Introduction

The extrusion of a viscoelastic jet from a die into an inviscid medium is an important situation occurring in polymer processing applications; see, for example, Tanner [33]. It is commonly referred to as the die-swell or extrudate-swell problem. The die may be a cylindrical pipe or a planar channel. Two characteristics of the die-swell problem are the expansion of the jet and the presence of a stress singularity at the exit of the die. The swelling of the extrudate for a viscoelastic fluid can be significantly more than that in the Newtonian case, see Tanner [32]. The presence of the stress singularity arises from the abrupt change in boundary conditions at the die exit. Its determination is crucial for understanding the extrudate-swell phenomenon as discussed by, for example, Andre and Clermont [1] and Tanner [32,34].

A simplified version of the die-swell problem is the so called stick-slip problem. Here the free surface is now fixed as a smooth continuation of the die wall with the swelling effect suppressed. Tanner and Huang [35] describe its possible setup through consideration of a repeating pattern of equally spaced channel walls. It is a situation in which the stress singularity at the die lip can be investigated and may be regarded as a first step toward understanding the more involved die-swell problem. It is emphasised that the term stick-slip is used here in regard to the change in the boundary conditions as the fluid leaves the pipe/channel and not to experimentally observed spurt flow with the extrudate exhibiting alternate smooth and sharskin regions, see, for example Denn [5].

In the Newtonian case, the stick-slip problem for Stokes flow (absence of inertia) was completely solved by Richardson [27] in the planar case and Trogdon and Joseph [36] in the 3-d axisymmetric case. For Newtonian fluids it may be considered to arise in the limit of large surface tension. Both sets of authors exploited the problem linearity and strip geometry by using the Weiner–Hopf technique, with in addition Trogdon and Joseph showing consistency with a matched eigenfunction expansion approach. The more general die-swell problem for a Newtonian fluid, has been considered analytically by Solonnikov [31].

For viscoelastic fluids, there is a paucity of analytical results and the question of well-posedness for these problems is an open issue. Further, numerical simulation tends to be problematic, see for example Lipscombe et al. [20] and Fortin et al. [10] for difficulties encountered in earlier numerical work. This has been attributed to the highly singular stresses encountered. Consequently both numerical and analytical work near the singularity has seen either the modification of the viscoelastic constitutive equations or the introduction of slip on the die walls. For example, Apelian et al. [2] and King et al. [18] use the Modified UCM model in place of UCM or Oldroyd-B models, whilst slip on the die walls has been used by Salamon et al. [28] for the Oldroyd-B model (and Silliman and Scriven [30] for a Newtonian fluid). A comprehensive summary

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of the schemes and viscoelastic models simulated for stick-slip and die-swell can be found in Ngamaramvaranggul and Webster [24] and more recently in Karapetsas and Tsamopoulos [16,17]. Analytically, Tanner and Huang [35] used an adaption of the J-integral approach from fracture mechanics to deduce that the singularity behaviour for Phan-Thien–Tanner (PTT), Modified Upper Convected Maxwell Model (MUCM) and general network models were of Newtonian form. Nothing definitive could be said for UCM and Oldroyd-B flows. (The approach usefully allowed the singularity intensity factors to be deduced for Newtonian and generalised Newtonian (particularly power law) fluids). Fontelos and Friedman [9] obtained existence and uniqueness results for a class of Oldroyd models (that do not include the B and A variants) in stick-slip.

Our focus here will be determining the stress singularity at the die exit for stick-slip flow of the Giesekus viscoelastic model. The Giesekus model [12,13], is a class of constitutive equations based on anisotropic drag and the concept of a deformation dependent tensorial mobility of dissolved molecules. It describes how the relaxation time of a molecule (elastic dumbbell) is altered when the surrounding molecules (elastic dumbbells) are oriented. The relaxation behaviour becomes anisotropic and results in an additional quadratic term of the stress tensor compared to the Maxwell model. A better description of polymeric solutions and melts is obtained, than for some other rheological models such as the Oldroyd-B model or corotational model. It enables a qualitative description of a number of well-known properties of viscoelastic fluids, namely shear thinning, non-zero second normal stress coefficient and stress overshoot in transient shear flows; see Giesekus [14], Larson [19] and Bris et al. [4].

Currently, the Giesekus model has not received attention within such an analytical study. The approach will use the method of matched asymptotic expansions that was successfully use by Evans [8] for the affine PTT model. It may be anticipated that its behaviour should be similar to the PTT model, since both involve quadratic stress terms. The main results of the paper will show that on small radial distances r near to the singularity:

1. The stress field is Newtonian dominated. Away from the stick and slip surfaces, the solvent stresses thus dominate and are $O(r^{-\frac{1}{2}})$ whilst the polymer stresses are $O(r^{-\frac{5}{16}})$ (which compare to $O(r^{-\frac{4}{11}})$ for PTT).
2. A boundary layer of thickness $O(r^{\frac{5}{4}})$ is required at the stick surface to accommodate viscometric flow. This thickness should be compared with $O(r^{\frac{7}{6}})$ for PTT.
3. A boundary layer of thickness $O(r^{\frac{17}{14}})$ is required at the slip surface to arrest elongational growth of the stresses. This thickness compares with $O(r^{\frac{23}{20}})$ for PTT.

Thus the polymer stress is less singular than that obtained for PTT, but the boundary layers are correspondingly narrower than their PTT counterparts. This is a trend that was identified for the high Weissenberg number boundary layers of Hagen and Renardy [15] and re-entrant corner behaviour discussed in Evans [6,7]. Crucial to these results is the presence of a solvent viscosity and the quadratic stress terms. The solvent viscosity has a regularizing effect on the model behaviour, with the polymer stresses being less singular than the solvent stresses. The presence of the quadratic stress terms arrest the strong stress growth that occurs in elongational flow after the die exit. The loss of either of these effects from the model is sufficient to significantly change the asymptotic behaviour at the singularity, which currently remains unknown in these limits of the model.

The advantages of determining the stress singularity are several. First it is a test of the rheology, to see how the constitutive equations behave under large stresses. Second, the form of the singularity is of use to numerical schemes, where it's behaviour can be incorporated to improve accuracy. This is particularly important for viscoelastic models which have strong hyperbolic properties that tend to propagate inaccuracies along streamlines. This has successfully been done for Newtonian fluids, where Georgiou et al. [11] introduced singular finite elements in the vicinity of the singularity to improve the solution accuracy and speed up the rate of convergence. However, this approach relies upon knowing the analytical form of the singularity. Thirdly, it adds to a catalogue of reference behaviours.

The problem formulation is introduced in Section 2, where the governing equations, boundary conditions and their non-dimensionalisation is detailed. The details of the asymptotic analysis are then given in Section 3. The analysis is performed in both the Cartesian and natural stress formulations of the constitutive equations. The most efficient approach for the analysis is using natural stress variables, where the link between solutions in the asymptotic regions occurs at leading order. However, performing the analysis in Cartesian variables is useful as it provides a consistency check on the natural stress results and is arguably easier to interpret physically particularly near the boundaries. However, it does suffer from requiring higher order terms in the outer expansions to communicate the correct stress information between the stick and slip surfaces. It is thus advantageous to record the details for both formulations. Finally in Section 4 a summary of the results in dimensional form is given.

2. Problem formulation

The geometry for classical stick-slip flow is depicted in Fig. 1 for the planar channel case. The channel width is taken as $2H$, with an assumed incoming plane Poiseuille flow far upstream with mean speed V. The fluid exits the channel at $x = 0$, far downstream of which it has a fully developed (shear-free) plug flow. If we take the speed of the plug flow as V, then the Poiseuille flow takes the form

$$\mathbf{v} = \left(3V\frac{y}{H}\left(1-\frac{y}{2H}\right), 0\right) \tag{2.1}$$

which follows from mass conservation through a flux balance for the two flows. The reverse flow set-up of slip-stick will also be considered. This being more for mathematical interest rather than practical.

The governing equations for steady incompressible planar flow of the Giesekus fluid are written in dimensional slow flow form

$$\nabla\cdot\mathbf{v} = 0, \quad 0 = -\nabla p + \nabla\cdot\boldsymbol{\tau}, \tag{2.2}$$

where $\mathbf{v} = (u, v)^T$ is the velocity field (represented by the usual 2-D stream function ψ) and p the pressure. The extra stress tensor $\boldsymbol{\tau} = \boldsymbol{\tau}^s + \boldsymbol{\tau}^p$ consists of a Newtonian solvent contribution $\boldsymbol{\tau}^s$ and an elastic polymeric contribution $\boldsymbol{\tau}^p$. The solvent stress is given by

$$\boldsymbol{\tau}^s = 2\eta_s\mathbf{D}, \tag{2.3}$$

where η_s is the solvent viscosity and $\mathbf{D}$ is the rate of strain (or deformation rate) tensor given by

$$\mathbf{D} = \frac{1}{2}(\nabla\mathbf{v} + (\nabla\mathbf{v})^T). \tag{2.4}$$

The extra elastic stress tensor $\boldsymbol{\tau}^p$ is taken to satisfy the Giesekus constitutive equation

$$\boldsymbol{\tau}^p + \lambda\left(\overset{\triangledown}{\boldsymbol{\tau}^p} + \frac{\alpha_{\text{mob}}}{\eta_p}(\boldsymbol{\tau}^p)^2\right) = 2\eta_p\mathbf{D}, \tag{2.5}$$

where λ is the stress relaxation time, α_{mob} is the mobility parameter of the model, η_p the polymer viscosity and the upper convected derivative of the elastic stress being

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