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Bubble rise dynamics in a viscoplastic material

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ABSTRACT

The axisymmetric dynamics of a bubble rising in a Bingham fluid under the action of buoyancy is studied. The Volume-of-Fluid (VOF) method is used to solve the equations of mass and momentum conservation, coupled to an equation for the volume fraction of the Bingham fluid. A regularised constitutive model is used for the description of the viscoplastic behavior of the material. The numerical results demonstrate that the rise dynamics are complex for large yield stresses, and for weak surface tension. Under these conditions, for which the bubble is highly deformable, the rise is unsteady and is punctuated by periods of rapid acceleration which separate stages of quasi-steady motion. During the acceleration periods, the bubble aspect ratio exhibits oscillations about unity, whose amplitude and wavelength increase with increasing yield stress and decreasing surface tension. These oscillations are accompanied by the alternating formation and destruction of unyielded zones at the bubble equator, as the bubble appears to "swim" upwards.

1. Introduction

The motion of droplets in fluids that exhibit yield stress is important in many engineering applications, including food processing, oil extraction, waste processing and biochemical reactors. Yield stress fluids or viscoplastic materials flow like liquids when subjected to stress beyond some critical value, the so-called yield stress, but behave as a solid below this critical level of stress; detailed review on yield stress fluids can be found in [1,2]. As a result the gravity-driven bubble rise in a viscoplastic material is not always possible as in the case of Newtonian fluids but occurs only if buoyancy is sufficient to overcome the material's yield stress [3,4]; the situation is also similar for the case of a settling drop or solid particle [5].

The first constitutive law proposed to describe this material behavior is the Bingham model [6] which was later extended by Herschel and Bulkley [7] to take into account the effects of shear-thinning (or thickening). According to this model the material can be in two possible states; it can be either yielded or unyielded, depending on the level of stress it experiences. As the common boundary of the two distinct regions the so-called yield surface is approached, the exact Bingham model becomes singular. In simple flows this singularity does not generate a problem, but, in more complex flows the discontinuous behavior of the Bingham model may pose significant difficulties due to the fact that in most cases the yield surface is not known *a priori* but must be determined as part of the solution. Nevertheless, there are examples of successful analysis of two-dimensional flows using this model at the expense of relatively complicated numerical algorithms [5,8–10]. A simpler way to overcome these difficulties is to modify the Bingham constitutive equation in order to produce a non-singular constitutive law, by introducing a 'regularization' parameter [11]. This method has been used with success by several researchers in the past [4,12–15] and when used with caution can give significant insight in the behavior of viscoplastic materials.

The motion of air bubbles in viscoplastic materials has attracted the interest of many research groups in the past. The first reported experimental study on rising bubbles in a viscoplastic material was done by Astarita and Apuzzo [16] who reported bubble shapes and velocities in Carbopol solutions. They observed that curves of bubble velocity vs bubble volume for viscoplastic liquids had an abrupt change in slope at a critical value of bubble volume that depended on the concentration of Carbopol in the solution, i.e. the yield stress of the material. Many years later, Terasaka and Tsuge [17] used xanthan gum and Carbopol solutions to examine the formation of bubbles at a nozzle and derived an approximate model for bubble growth. Dubash and Frigaard [18] also performed experiments with Carbopol solutions and were able to comfirm the observations of Astarita and Apuzzo [16] on the existence of a critical bubble radius required to set it in motion and noted that the entrapment

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conditions are affected significantly by surface tension. It is also noteworthy that the observed bubble shapes inside a vertical pipe were different from [16] exhibiting a cusped tail, resembling much the inverted teardrop shapes often found inside a viscoelastic medium [16,19,20]. Similar bubble shapes have been found in the experimental studies by Sikorski et al. [21] and Mougin et al. [22], using Carbopol solutions of different concentrations. The latter authors also studied the significant role of internal trapped stresses within a Carbopol gel on the trajectory and shape of the bubbles; their findings were in agreement with an earlier study presented by Piau [23].

From a theoretical point of view, Bhavaraju et al. [24] performed a perturbation analysis in the limit of small yield stress for a spherical air bubble. Stein and Buggish [25] were interested on the mobilization of bubbles by setting an oscillating external pressure and provided analytical solutions along with some experimental data; the latter suggested that larger bubbles tend to rise faster than smaller bubbles at similar amplitudes. Dubash and Frigaard [3] employed a variational method to estimate the conditions under which bubbles should remain static. These estimations, however, were characterized as conservative, in the sense that they provide a sufficient but not necessary condition. A detailed numerical study of the steady bubble rise, using the regularized Papanastasiou model [26], has been performed by Tsamopoulos et al. [4]. These authors presented mappings of bubble and yield surface shapes for a wide range of dimensionless parameters, taking into account the effects of inertia, surface tension and gravity. Moreover, they were able to evaluate the conditions for bubble entrapment. Their work was followed by the study of Dimakopoulos et al. [10] who used the augmented Lagrangian method to obtain a more accurate estimation of the stopping conditions. It was shown that the critical Bingham number, Bn, does not depend on the Archimedes number in accordance with Tsamopoulos et al. [4], but depends non-monotonically on surface tension. We should note that in both studies the shape of the bubble near critical conditions could not reproduce the inverted teardrop shapes seen in experiments [18,21,22] and raised questions whether this is due to elasticity, thixotropy or wall effects. Besides the steady solutions it is also interesting to investigate the bubble dynamics through time-dependent simulations. This was done by Potapov et al. [27] and Singh and Denn [28] using the VOF method and the level-set method, respectively. Singh and Denn [28] considered creeping flow conditions and performed simulations for single and multiple bubbles. It was shown that multiple bubbles and droplets can move inside the viscoplastic material under conditions that a single bubble or droplet with similar properties would have been trapped unable to overcome the yield stress. Potapov et al. [27] also studied the case of a single or two interacting bubbles but also took into account the effect of inertia, albeit for a low Reynolds number. For the parameter range that they have used the single bubble always reached a quasi-steady state. We should note at this point that for some cases (e.g. for high values of the Archimedes number) Tsamopoulos et al. [4] were not able to calculate steady shapes which is probably an indication that the flow may become time-dependent. Even in the cases where a steady solution could be obtained it is not certain that this solution is stable. Therefore a question that arises is whether for some parameter values it is possible to get a time-dependent solution and what would be the dynamics of the bubble flow in this case. This is the question that our paper will attempt to address.

In this paper, we study the buoyancy-driven rise of a bubble inside an infinite viscoplastic medium, assuming axial symmetry. To account for the viscoplacity we consider the regularized Herschel-Bulkley model. We employ the Volume-of-Fluid method to follow the deforming bubble along the domain. Our results indicate that in the presence of inertia and in the case of weak surface tension the bubble does not reach a steady state and the dynamics may become complex for sufficiently high yield stress of the material.

The rest of the paper is organized as follows. In Section 2, we outline the governing equations, and in Section 3 discuss our numerical results. Finally, concluding remarks are given in Section 4.

2. Formulation

2.1. Governing equations

We consider the rise of a bubble (Newtonian fluid ‘B’) in a viscoplastic material (fluid ‘A’) under the action of buoyancy within a cylindrical domain of diameter H and height L, as shown in Fig. 1. We use an axisymmetric, cylindrical coordinate system, (r, z), to model the flow dynamics, in which r and z denote the radial and axial coordinates, respectively, the latter being aligned in the opposite direction to gravity. The bubble is initially present at a distance z_i above the bottom of the domain located at $z = 0$. The governing equations of the problem correspond to those of mass and momentum conservation:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right] = -\nabla p + \nabla\cdot\left[\mu(\nabla\mathbf{u} + \nabla\mathbf{u}^T)\right] + \mathbf{F}, \tag{2}$$

and the following equation for the volume fraction, c, of the fluid A.

$$\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c = 0. \tag{3}$$

In the above equations, $\mathbf{u} = (u_r, u_z)$ denotes the velocity field in which u_r and u_z represent the radial and axial velocity components, respectively, p is the pressure field, t denotes time, and $\mathbf{F}$ is the combined body and surface forces per unit volume, which include the gravity and surface tension forces given by:

$$\mathbf{F} = \delta\sigma\kappa\mathbf{n} - \rho g\mathbf{j}; \tag{4}$$

here, j denotes the unit vector along the vertical direction, σ and g represent the (constant) interfacial tension and gravitational acceleration, respectively, δ is the Dirac delta function, and $\kappa = \nabla\cdot\mathbf{n}$ is

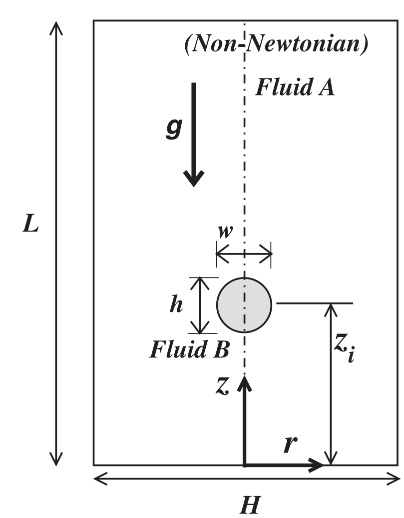

Fig. 1. Schematic diagram of a bubble of fluid ‘B’ rising inside a Bingham fluid ‘A’ under the action of buoyancy. The bubble is placed at $z = z_i$; the value of H, L and z_i are taken to be $20R$, $48R$, and $10.5R$, respectively. Initially the aspect ratio of the bubble, h/w is 1, wherein h and w are the maximum height and width of the bubble.

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