Journal of Non-Newtonian Fluid Mechanics 214 (2014) 57–68

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: http://www.elsevier.com/locate/jnnfm

A new viscoelastic benchmark flow: Stationary bifurcation in a cross-slot

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ARTICLE INFO

Article history:
Received 10 October 2013
Received in revised form 26 September 2014
Accepted 29 September 2014
Available online 6 October 2014

Keywords:
Finite-volume method
Cross-slot flow
Elastic instabilities
Benchmark flow
Flow transitions

ABSTRACT

In this work we propose the cross-slot geometry as a candidate for a numerical benchmark flow problem for viscoelastic fluids. Extensive data of quantified accuracy is provided, obtained via Richardson extrapolation to the limit of infinite refinement using results for three different mesh resolutions, for the upper-convected Maxwell, Oldroyd-B and the linear form of the simplified Phan-Thien–Tanner constitutive models. Furthermore, we consider two types of flow geometry having either sharp or rounded corners, the latter with a radius of curvature equal to 5% of the channel's width. We show that for all models the inertialess steady symmetric flow may undergo a bifurcation to a steady asymmetric configuration, followed by a second transition to time-dependent flow, which is in qualitative agreement with previous experimental observations for low Reynolds number flows. The critical Deborah number for both transitions is quantified and a set of standard parameters is proposed for benchmarking purposes.

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1. Introduction

Arratia et al. [1] demonstrated experimentally that the low-Reynolds number (*Re*) flow of a flexible polymer solution (a polyacrylamide aqueous solution) through a microfluidic cross-slot geometry can give rise to two different instabilities: a first instability, in which the flow remains steady but becomes spatially asymmetric; and a second instability, in which the flow becomes unsteady and fluctuates with time. In fact, such asymmetries, which are purely-elastic in nature, are not new and were also present in the early experimental work of Gardner et al. [2]. Although the photon-correlation velocimetry technique used by Gardner and co-workers was rather noisy, their velocity profiles are clearly asymmetric downstream of the cross-slot. Gardner et al. [2] thought that the asymmetry was a consequence of either imperfections in the geometry or a "fluidic-type" instability. These experimental results were the stimuli for our own numerical investigations. In Poole et al. [3], a finite-volume numerical technique was used to show that such asymmetries could be predicted even for the upper-convected Maxwell (UCM) model in a truly-inertialess flow ($Re \to 0$), and that they were probably a consequence of the high compressive normal stresses developed by the viscoelastic fluid when the two incoming streams join. These normal stresses lead to a concave velocity profile, together with an inflection point, as illustrated in [3] along the diagonal lines $y = \pm x$. It is well known that inflection points in velocity profiles are a necessary condition for inviscid instabilities, but even with viscous fluids inflection points are often associated with flow instabilities, at least in flows with non-negligible inertia (a famous example are the experiments and theoretical predictions of G.I. Taylor in rotating Couette flows [4] – now known as Taylor–Couette flows). However, this association is empirical at best, lacking the mathematical/physical insight quality of the Rayleigh criterion for inviscid instabilities, and should thus be regarded with caution.

In this work, we propose the two-dimensional (2D) cross-slot geometry, illustrated schematically in Fig. 1, as an interesting and useful candidate for a numerical benchmark case for viscoelastic fluid flows. For a wide range of differential viscoelastic models, namely the UCM and Oldroyd-B models [5] and the Phan-Thien and Tanner (PTT) model [6], we show that, in the limit of negligible inertia, *i.e.* when *Re* approaches zero, the flow exhibits two different types of purely-elastic instabilities. Above a first critical value of the Deborah number, $De = \lambda U/D = \lambda Q/D^2$, where λ is the fluid relaxation time, U the bulk velocity in each arm, Q the flow rate per unit depth in each arm and D the channel width, the steady flow becomes spatially asymmetric, even though the geometry is perfectly symmetric; at higher *De* the flow then becomes time-dependent. The geometry is simple and the steady asymmetric flow is well defined. The local evolution of the bifurcation parameter *DQ* [3] is well described by a square-root fit which is typical of supercritical pitchfork bifurcations, $DQ = A\sqrt{De - De_{CR}}$, and the

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http://dx.doi.org/10.1016/j.jnnfm.2014.09.015
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critical Deborah number, De_{CR}, can be predicted with high accuracy, based on the Richardson extrapolation of results for systematically-refined meshes. In addition the flow contains an internal stagnation point where a fluid element is subjected to large extensional stresses. Such behaviour allows the accuracy of numerical methods to be tested away from any influence of boundary conditions, namely by analysis of the mesh-wise convergence of the local Weissenberg number, $Wi_o = \lambda\dot{\varepsilon}_o$, calculated at the stagnation point. Indeed, rounding the corners of the cross-slot with a small radius of curvature seems to have little to no influence on the pitchfork bifurcation, as will be shown in the results section.

Up until the work by Arratia et al. [1], although a great deal of research had been conducted using stagnation point geometries similar to the cross-slot device, such as the four roll mill and opposed-jet devices, such steady asymmetries have largely gone unreported. Therefore, we do not present a detailed review of the early literature here. The interested reader is referred to Schoonen et al. [7] and Remmelgas et al. [8]. For in-depth reviews of the early literature on elastic instabilities see [9,10]. In numerical studies the inability to predict steady asymmetric flow is easily explained, since most studies imposed symmetry boundary conditions to reduce the computational burden (*e.g.* [7,8]). Experimentally it is likely that the main reason why such instabilities have not been observed is the relative unimportance of elastic effects in macro-sized devices, when compared to their micro-sized counterparts (*cf.* definition of *De*). The microfluidic nature of the experiments of both Arratia et al. [1] and Gardner et al. [2], in which effects due to elasticity are inherently enhanced given the small length scales of the flow, is the main reason why such instabilities became observable. Of course the paper by Gardner and co-workers predates the term "microfluidics" by some decades, but their channel width, $D = 250\ \mu m$, is actually *smaller* than in the work of Arratia et al., at $D = 650\ \mu m$. This argument is further supported by experimental results for surfactant flows, published by Pathak and Hudson [11] in the same year as Arratia et al. [1], again in a microfluidic geometry, in which the steady asymmetry and subsequent transition to time-dependent flow were also observed. Since 2006, other publications have reported steady-state asymmetries in cross-slot flows of wormlike micellar solutions, including fully asymmetric flow [11,12], *Wi-Re* stability diagrams delimiting different flow behaviours [12,13] and also the formation of lip vortices in the inlet channels for strongly viscoelastic fluids [12,14].

With the increasing prevalence of microfluidic devices in viscoelastic fluid flow research and engineering, in recent years a number of publications have addressed the issue of steady asymmetries in various cross-slot flows of polymeric solutions. Rocha et al. [15] simulated FENE-CR and FENE-P fluid flows and predicted a transition to steady asymmetry at progressively lower *De* with increasing values of the extensibility parameter and polymer concentration. The effect of corner sharpness was not significant up to a radius of curvature of 50% of the channel width. According to Larson et al. [16] and later McKinley et al. [17], streamline curvature is required for the onset of elastic instabilities. This curvature may either be intrinsic to the geometric confinement

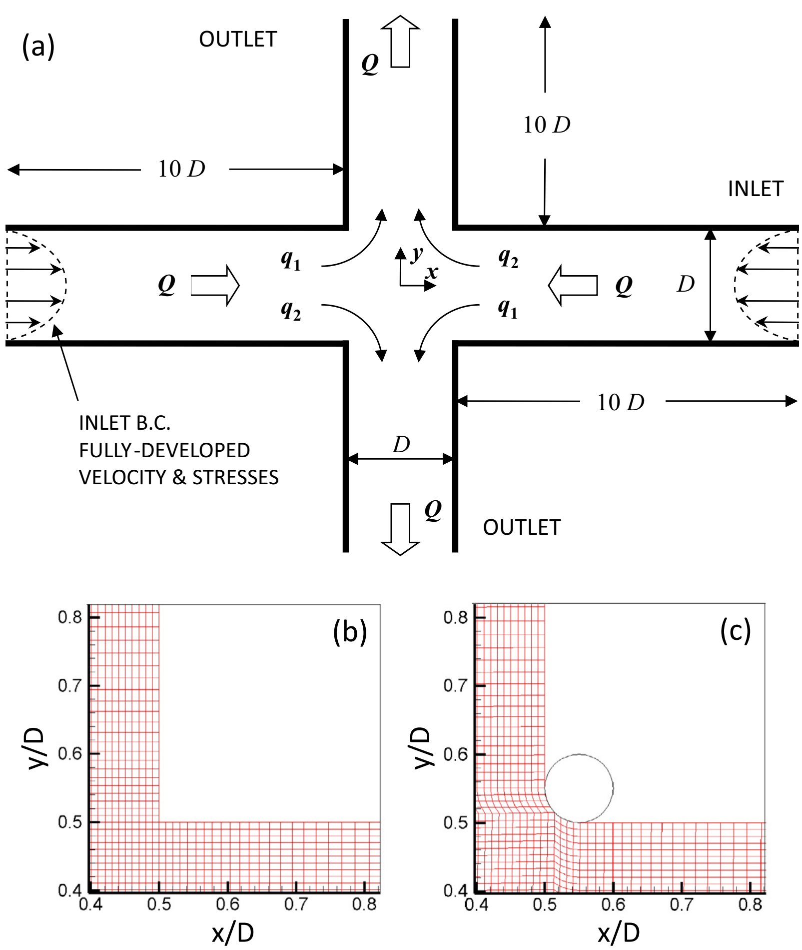

Fig. 1. (a) Schematics of the cross-slot geometry. Zoomed view of the meshes near the corners of the (b) sharp and (c) rounded geometries, for meshes M2-R0 and M2-R5, respectively (see Table 1 for details). The rounded corner has a radius of curvature of 5% of the channel width, as illustrated by the superimposed circle in (c).

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