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The effect of a variable plastic viscosity on the restart problem of pipelines filled with gelled waxy crude oils

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ABSTRACT

The effect of a variable plastic viscosity is numerically investigated on the success of the restart operation for a pipeline filled with fully-gelled waxy crude oil. To investigate the separate effects of structure- and shear-dependent viscosity, waxy crude oil is assumed to obey the Houska rheological model. In order to precisely capture the shape and position of the yielding surface, a variational approach is used to formulate the restart problem for this particular fluid model. The numerical results show that a variable plastic viscosity has a significant effect on the restart operation. For certain set of parameters the restart operation is shown to fail if the plastic viscosity is constant but is successful if the plastic viscosity is (moderately) structure-dependent. However, the time needed by the liquefied gel to discharge from the pipe outlet section is increased if the plastic viscosity is structure-dependent. Surprisingly, the shear-thinning behavior of waxy crude oil is predicted to lower the (steady) flow rate even when the restart is successful.

1. Introduction

Pipelines are generally regarded as one of the most economical means for the transportation of crude oils. This is certainly true for conventional (light) oils, but for heavy crudes the operation is often realized to be a difficult task [1]. This is particularly so if the heavy oil contains significant amount of wax (say, larger than 4%). That is to say that, at temperatures lowers than the wax-appearance-temperature (WAT) the wax, normally dissolved in the oil, starts to crystallize. The paraffin crystals form an interlocking gel-like structure which can potentially block the pipeline under severe situations. On the other hand, at temperatures slightly below WAT the oil exhibits a variety of non-Newtonian behavior comprising non-zero yield stress, shear-thinning viscosity, time-dependency, temperature-dependency, and viscoelasticity [2–6]. (It also becomes weakly compressible as a result of gas voids formed during thermal shrinkage.) The complex rheology of waxy crude oils makes a theoretical analysis of their pipe flow a difficult task. One can notably mention the restart problem of pipelines filled with gelled oil—a situation encountered quite frequently when a pipeline is shut-down for maintenance or emergency reasons. Predicting the pressure needed for a successful restart is an important but challenging task in oil industry [2–6].

Over the past two decades, there have been several efforts to investigate the effect of physical/rheological properties of waxy crude oils on the pressure required to restart pipelines filled with their gel. For ease of analysis, most studies carried out in the past have relied on simple rheological models and/or certain approximations. Vinay et al. [7–9] relied on the Bingham model for this purpose and showed that the pressure needed for the restart strongly depends on the yield stress and its temperature-dependency. They also showed that the driving pressure decreases if the compressibility of the gel is taken into account. In another interesting work, Chang et al. [10], and Davidson et al. [11] relied on a time-dependent Bingham model and showed that a time-dependent yield stress delays the restart process. Wachs et al. [12] resorted to the robust Houska model for representing the viscoplastic/thixotropic nature of waxy crude oils [13,14] and showed that variation of yield stress in the radial direction plays a key role in the success of restart operation. They also showed that there exist situations where the pressure drop required for pipeline restart is lower than the theoretical minimum pressure drop (which is equal to $\Delta p = 4\tau_y \frac{L}{D}$ where τ_y is the yield stress, L is the length of the pipeline, and D is the pipe diameter) but the flow still restarts thanks to a combined effect of thixotropy and compressibility.

To simplify the analysis, Wachs et al. [12] assumed that the plastic viscosity of waxy crude oil is constant. In practice, however, the plastic viscosity is known to be both structure- and

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shear-dependent. It is the main objective of the present work to show that variation of the plastic viscosity with shear or structure plays an important role in the success of any restart attempt. To that end, like Ref. [12] we ignore the velocity component in the radial direction but allow the axial velocity to vary in both the axial and radial directions—the so-called 1.5D analysis [12]. To reach its objectives the work is organized as follows: in Section 2 the governing equations are presented briefly together with their pertinent boundary conditions. The numerical method of solution (which is based on a variational framework together with Uzawa's algorithm) will be described next. Numerical results are then presented addressing the effect of a variable plastic viscosity on the restart problem. The work is concluded by highlighting its major findings.

2. Governing equations

We consider the restart problem for a very long pipeline of length L and diameter D initially filled with a weakly-compressible fully-gelled waxy crude oil (see Fig. 1). A constant (negative) pressure gradient is suddenly applied at the pipe inlet to break the gel and restart the flow. Our main objective is to predict this pressure gradient and see how it is affected by the fluid's rheology. To that end, we start with presenting the governing equations in its most general form. Assuming isothermal conditions, the governing equations comprise the Cauchy equations of motion together with the continuity equation,

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\underset{\sim}{\tau} \tag{1a}$$

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{u}) = 0 \tag{1b}$$

where $\mathbf{u}$ is the velocity vector, ρ is the density, p is the isotropic pressure, and $\underset{\sim}{\tau}$ is the extra stress tensor. To account for the weakly compressible nature of gelled oil, the continuity equation is re-written as,

$$c_0\left(\frac{\partial p}{\partial t}+\mathbf{u}\cdot\nabla p\right) + \nabla\cdot\mathbf{u} = 0 \tag{1c}$$

where c_0 is the isothermal compressibility factor defined by: $c_0 = \frac{1}{\rho}(\partial\rho/\partial p)_T$ with the subscript T denoting the temperature. In this study, we rely on the Houska rheological model for representing viscoplastic, thixotropic, and shear-thinning behavior of waxy crude oils. For this rheological model we have [13,14],

$$\underset{\sim}{\tau} = \begin{cases} 2[k+\lambda\Delta k]\dot{\gamma}^{(n-1)}\left(\underset{\sim}{D}-\frac{1}{3}(\nabla.\mathbf{u})\underset{\sim}{I}\right)+\left[\tau_{y,0}+\lambda(\tau_{y,1}-\tau_{y,0})\right]\frac{\underset{\sim}{D}}{\|\underset{\sim}{D}\|}; & \text{if } \|\underset{\sim}{\tau}\| \geqslant \tau_{y,0}+\lambda(\tau_{y,1}-\tau_{y,0}) \\ \underset{\sim}{D}=0; & \text{if } \|\underset{\sim}{\tau}\| < \tau_{y,0}+\lambda(\tau_{y,1}-\tau_{y,0}) \end{cases} \tag{2}$$

where $\underset{\sim}{I}$ is the identity tensor, $\underset{\sim}{D}$ is the rate-of-deformation tensor, and $\dot{\gamma} = 2\|\underset{\sim}{D}\|$ is the shear rate with the norm $\|\underset{\sim}{D}\|$ defined by,

$$\|\underset{\sim}{D}\| = \left(\frac{1}{2}\sum_{i,j}|D_{ij}|^2\right)^{1/2}. \tag{3}$$

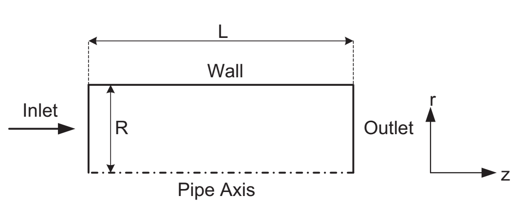

Fig. 1. Physical domain of the problem.

In Eq. (2), λ is the structural parameter which lies in the range $[0, 1]$ with 0 meaning that the structure is fully broken down and 1 meaning that the material is fully gelled. The yield stress corresponding to these two limiting cases have been denoted by τ_{y0} and τ_{y1}, respectively. Similarly, in Eq. (2) fluid's "consistency index" corresponding to these limiting cases have been denoted by k and $k + \Delta k$ (with "n" serving as the power-law exponent). For fluids obeying the Houska model, the structural parameter satisfies a kinetic equation of the form [13,14],

$$\frac{\partial \lambda}{\partial t}+\mathbf{u}\cdot\nabla\lambda = a(1-\lambda) - b\lambda\dot{\gamma}^m, \tag{4}$$

where "a" controls the build-up mechanism, and "b" controls the break-up mechanism (which is itself allowed to be slightly shear-dependent with "m" controlling the degree of its shear-dependency). It should be noted that the eight material properties appearing in the Houska model (i.e., a, b, m, n, $\tau_{y,0}$, $\tau_{y,1}$, k, and $k + \Delta k$) are determined by curve-fitting to the rheological data. Now, to close the problem it suffices to decide on the boundary conditions. Based on Fig. 1, in cylindrical coordinate system, the physical boundary conditions are:

- At the pipe inlet: $u = 0$, $p = p_{in}$, $\tau_{zz} = 0$.
- At the pipe outlet: $u = 0$, $p = p_{out}$, $\tau_{zz} = 0$.
- At the pipe wall: $u = 0$, $w = 0$.
- At the pipe centerline: $u = 0$, $\tau_{rz} = 0$.

where "u" is the radial velocity, and "w" is the axial velocity—the tangential velocity, v, is zero for axisymmetric flows. In order to work with dimensionless parameters, we substitute:

$$\begin{aligned} &r^* = \frac{r}{R}, \quad z^* = \frac{z}{R}, \quad t^* = \frac{t}{R/w_0} \quad \rho^* = \frac{\rho}{\rho_0} \quad \mu^* = \frac{\mu_p}{\mu_r}, \\ &u^* = \frac{u}{w_0} \quad w^* = \frac{w}{w_0} \quad p^* = \frac{p}{R\left(\frac{\Delta p}{L}\right)} \quad \tau^*_{rz} = \frac{\tau_{rz}}{R\left(\frac{\Delta p}{L}\right)} \quad \tau^*_{zz} = \frac{\tau_{zz}}{R\left(\frac{\Delta p}{L}\right)} \end{aligned} \tag{5}$$

where ρ_0 is the density at the pipe outlet, μ_p is the plastic viscosity, μ_r is the reference viscosity, and $w_0 = \frac{R^2}{\mu_r}\frac{\Delta p}{L}$ is the velocity scale ($\Delta p = p_{in} - p_{out}$). For Houska fluids, we can rely on either the plastic viscosity corresponding to $\lambda = 0$ (denoted by μ_o) or $\lambda = 1$ (denoted by μ_1) as the reference viscosity. Therefore, we are left with the option to choose between $\mu_0 = k(w_0/R)^{n-1}$ and $\mu_1 = (k + \Delta k)(w_0/R)^{n-1}$, for this purpose. Wachs et al. [12] relied on the plastic viscosity corresponding to $\lambda = 0$ as the reference viscosity. But, because they considered the case of $n = 1$ only, it did not really matter which viscosity they should have used as the reference viscosity. In our opinion, the plastic viscosity corresponding to $\lambda = 1$ is more suitable for this purpose because in restart problems the structure is initially complete and the viscosity is equal to μ_1. But, because in the present work we are not going to rely on any scaling arguments for simplifying the governing equations, our choice for the reference viscosity is dictated only by the numerical difficulties which we might encounter in the course of the computations. That is to say that, we have at our disposal to rely on either μ_0 or μ_1 as the reference viscosity whenever the need arises. It needs to be mentioned that regardless of which viscosity scale we use in practice, we always end up with the tools required to investigate the effect of a variable plastic viscosity on the characteristics of the restart process. With this in mind, Table 1 presents the definition of all dimensionless numbers pertinent to the restart problem of Houska fluids.

The problem as posed above is too general to render itself to a numerical solution at a reasonable cost. To simplify the problem, Wachs et al. [12] assumed that the velocity component in the radial direction is small compared to that in the axial direction (i.e., $u << w$). With this assumption the r-momentum equation is reduced simply to $\partial p/\partial r \approx 0$. The set of z-momentum and continuity equations are also simplified to [12]: