Journal of Non-Newtonian Fluid Mechanics 201 (2013) 10–16

Contents lists available at SciVerse ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: http://www.elsevier.com/locate/jnnfm

Viscoelastic secondary flows in serpentine channels ☆

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ARTICLE INFO

Article history:
Received 16 April 2013
Received in revised form 4 July 2013
Accepted 4 July 2013
Available online 15 July 2013

Keywords:
Finite volume method
Oldroyd-B model
UCM model
Secondary flow
Serpentine channel

ABSTRACT

We report the results of a detailed numerical investigation of inertialess viscoelastic fluid flow through three-dimensional serpentine (or wavy) channels of varying radius of curvature and aspect ratio using the Oldroyd-B model. The results reveal the existence of a secondary flow which is absent for the equivalent Newtonian fluid flow. The secondary flow arises due to the curvature of the geometry and the streamwise first normal–stress differences generated in the flowing fluid and can be thought of as the viscoelastic equivalent of Dean vortices. The effects of radius of curvature, aspect ratio and solvent-to-total viscosity ratio on the strength of the secondary flow are investigated. The secondary flow strength is shown to be a function of a modified Deborah number over a wide parameter range.

1. Introduction

It is well known that flows within pipes and ducts can give rise to secondary flows. In addition to the base primary flow in the streamwise direction, a secondary flow, albeit usually much weaker, can develop in the cross-stream direction. In the case of Newtonian fluids, so-called Dean vortices [1,2] appear in curved ducts or bends and are a consequence of flow inertia. Dean flow gives rise to a pair of vortices in the cross-section carrying flow from the inside to the outside of the bend across the centre and back around the edges. (NB: Malheiro et al. [3] have recently studied the effect of elasticity on such vortices). For Newtonian fluids in the absence of inertia, or in the absence of curvature, i.e. in straight pipes and ducts of uniform cross-section, there is no physical driving mechanism for a secondary flow and the laminar flow remains unidirectional. Interestingly, Lauga et al. [4] show that a secondary flow must develop if a channel has *both* varying cross-sectional area and non-constant curvature even in the creeping-flow limit. Note that turbulent flow can give rise to a secondary flow even in the case of straight ducts as long as the geometry is non-axisymmetric [5,6] i.e. not a circular pipe or a concentric annulus.

For viscoelastic fluid flows, in contrast, secondary flows can develop in the absence of inertia and curvature. Weak secondary flows are observed even for straight ducts of uniform cross-section as long as the geometry is non-axisymmetric. Such elastically-induced secondary flows are driven by imbalances in the second-normal stress difference and have been studied in detail by a number of authors [7–12]. Interestingly, Speziale [13] highlighted the relationship between this type of secondary flow and that driven by turbulence as discussed above. As the magnitude of the second-normal–stress difference is usually very small for most dilute polymer solutions, being estimated to be at most 20% of the first normal–stress difference for concentrated solutions and melts [14], these secondary flows tend to be extremely weak being of the order of 1% or less of the primary streamwise velocity [9]. As a consequence, numerical simulations of viscoelastic constitutive equations which predict a zero second normal–stress difference, such as the upper-convected Maxwell and Oldroyd-B models [15], the simplified Phan–Thien–Tanner (PTT) model [16] and FENE-type models [17,18], all predict unidirectional flow in such straight ducts, at least prior to the appearance of purely-elastic instabilities beyond a critical Weissenberg number [19].

The combined case of duct curvature and fluid elasticity in the inertialess limit, which has been significantly less studied, can also give rise to secondary flows. These secondary flows can occur in both axisymmetric as well as non-axisymmetric geometries and can be observed even for fluids which exhibit a zero second normal–stress difference in steady simple shear flow. This has been elegantly shown by Fan et al. [20] who investigated flow in curved pipes. To the best of our knowledge, for flows in non-axisymmetric

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0377-0257/$ - see front matter
http://dx.doi.org/10.1016/j.jnnfm.2013.07.001

geometries the only paper which investigates inertialess secondary flows is the recent work by Norouzi et al. [21]. They use the second-order fluid model [22] to investigate curved ducts with square cross-sections both with and without inertia. By varying the parameters in the second-order fluid model to control the ratio of first to second normal–stress differences, Norouzi et al. were able to show that the strength and direction of the secondary flow could be varied. When the first normal–stress difference was dominant the direction of the viscoelastic secondary flow was found to be in the same sense as that observed by Fan et al. [20], i.e. in the same sense as inertial Dean flow, but when the first normal–stress difference was zero and only second normal–stresses occurred the secondary flow changed direction. The second-order fluid used by Norouzi et al. is appropriate in the limit of vanishingly small elasticity, and therefore to small Deborah numbers, and is thus useful to investigate the qualitative behaviour of polymer flow phenomena such as the direction of secondary flows. The quantitative prediction of the strength of the secondary flow beyond the asymptotic limit of vanishingly elasticity will be influenced by the choice of constitutive equation and the effects of more realistic constitutive equations on the strength of this elastically-driven secondary flow has not been investigated. In the current paper we report the results of a detailed numerical investigation of inertialess viscoelastic fluid flow through three-dimensional serpentine (or wavy) channels [23,24] of varying radius and aspect ratios using the Oldroyd-B model to fully explore this secondary flow regime. Such serpentine channels are composed of a series of circular half loops of alternating curvature and represent prototype geometries for investigating curvature effects experimentally [23,24].

2. Viscoelastic constitutive equation and numerical method

The three-dimensional numerical simulations assume isothermal flow of an incompressible viscoelastic fluid described by the Oldroyd-B model [15] in a channel of rectangular cross section. The equations that need to be solved are those of mass conservation,

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

and momentum

$$\mathbf{0} = -\nabla p + \eta_s \nabla^2 \mathbf{u} + \nabla \cdot \boldsymbol{\tau}, \tag{2}$$

assuming creeping-flow conditions (i.e. the inertial terms are exactly zero), where $\mathbf{u}$ is the velocity vector with Cartesian components (u_x, u_y, u_z), p is the pressure and η_s is the solvent viscosity. For the Oldroyd-B model the evolution equation for the polymeric extra-stress tensor, $\boldsymbol{\tau}$, is

$$\boldsymbol{\tau} + \lambda\left(\frac{\partial \boldsymbol{\tau}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\tau}\right) = \eta_p(\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \lambda(\boldsymbol{\tau} \cdot \nabla \mathbf{u} + \nabla \mathbf{u}^T \cdot \boldsymbol{\tau}), \tag{3}$$

where λ and η_p are the relaxation time and polymeric contribution to the viscosity of the fluid respectively, both of which are constant in this model. For a large number of simulations shown here we set the solvent viscosity contribution to zero and, in this case, the upper-convected Maxwell (UCM) model is recovered.

Although the Oldroyd-B model exhibits an unbounded steady-state extensional viscosity above a critical strain rate ($1/2\lambda$), in shear-dominated serpentine channel geometries such model deficiencies are unimportant and it is arguably the simplest differential constitutive equation which can capture many aspects of highly-elastic flows [25,26]. Many more complex models (e.g. the FENE-P, Giesekus and Phan–Thien–Tanner models – see e.g. Bird et al. [22]), simplify to the Oldroyd-B model in certain parameter limits and thus its generality makes it an ideal candidate for fundamental studies of viscoelastic fluid flow behaviour. The governing equations are solved using a time-marching implicit finite-volume numerical method, based on the logarithm transformation of the conformation tensor [27]. Additional details about the numerical method can be found in Afonso et al. [28,29] and in other previous studies (e.g. [30,31]). For low Wi the numerical solution converges to a steady solution, which was assumed to occur when the L_2 norm of the residuals of all variables reached a tolerance of 10^{-6}. Beyond a critical Weissenberg number a time-dependent purely-elastic instability occurs [24]. The results in the current paper are restricted to Weissenberg numbers below the occurrence of this purely-elastic instability: thus the flow remains steady.

3. Flow geometry, dimensionless numbers and computational meshes

The serpentine channels used in this work consist of a series of half-loops of width W, height H and inner radius R as shown schematically in Fig. 1. Although the geometries are fully three-dimensional we impose a symmetry boundary condition on the xy-centreplane to reduce the computational burden. Limited simulations on the complete domain confirmed that, for the steady results shown here, the imposition of symmetry has no effect on the results. In all the results which follow the symmetry plane is highlighted by a dashed boundary (see Fig. 1c for example). The inner and outer walls are also indicated. A series of geometries were created such that the effects of radius (R/W) and aspect ratio ($a = W/H$) could be investigated in the range $1 \leqslant R/W \leqslant 7$ and $0.5 \leqslant W/H \leqslant 4$.

For all results shown in this work the Reynolds number is identically zero. The Weissenberg number is defined as $Wi = \lambda U/W$, where λ is the relaxation time of the fluid and U/W represents a characteristic shear rate based on the channel width W and the bulk velocity U in the channel. A Deborah number can be defined as $De = \lambda U/R$ based on the ratio of the relaxation time of the fluid and a characteristic residence time in each half loop ($\sim R/U$).

The number of full(half) loops in each geometry was fixed at two(four): tests with more loops gave identical results. The majority of data pertaining to the secondary flow will be presented at the bend in the first half loop (location A1 in Fig. 1b). For the current results, where the Deborah number remains always less than one, memory effects remain small and secondary flow data at subsequent loops (e.g. A3 or B1) are essentially identical to the first loop (in the least favourable case for example when $R/W = 1$, $W/H = 1$ and $Wi = 0.6$ the secondary flow strength, as measured by the maximum spanwise velocity, differs by just 0.7% between locations A1, A3, B1 and B3).

For all of the serpentine channels the computational domain was mapped using three orthogonal blocks, one straight inlet section of length 10 W, one block comprising four half loops of varying curvature and a final straight exit section also 10 W in length. The main characteristics of the meshes are provided in Table 1. The information in Table 1 includes the total number of cells in the meshes (NC) together with the number of control volumes in each direction (NX, NY and NZ) and the total number of degrees of freedom (DOF) of the computed variables. The cell sizes are uniform in the y- and z-directions and in the x-direction in the second block. In the inlet(exit) channels the cell spacing in the x-direction decreases as the cells move towards(away) from the block containing the half-loops. It is important to note that the x, y, z coordinate system is fixed in space but that we will refer always to the velocity component in the streamwise direction as u, in the wall normal or transverse direction as v and the velocity in the spanwise direction (z) as w. As a consequence the streamwise velocity component u for example is only aligned with the x-direction in the straight inlet and outlet channels (and at locations A2, A4/B0, B2). Thus