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Abstract

We analyze the 1D flow of a Bingham fluid between two differentially heated vertical plates, in the presence of a stabilizing vertical temperature gradient, imposed at the walls. The solution is parameterized by the Bingham number, B, and the stratification parameter Γ, and is surprisingly complex. When $B \geq B_{cr} = 1/16$ the fluid is unyielded everywhere and heat transfer is via pure conduction. We refer to this as a 1–plug solution. For $B \lesssim B_{cr}$, a perturbation solution shows that yielding starts at the walls and the centerline of the channel, breaking into two asymmetric counter-current streams and with a single plug in each stream (a 2–plug solution). We characterize the solution regimes in the Γ-B plane in terms of the number of plugs that are found. We identify the main characteristics of these solutions and provide data suitable for numerical benchmarking. For increasing Γ and decreasing B, we show that in principle, an arbitrarily large number of plugs can be found in the finite width channel. Primarily we solve for the 1–plug (conductive), 2–plug and 3–plug solutions, which are found to dominate the Γ-B parameter space.

Keywords: Bingham Fluid, Natural Convection, Analytical Solution

1. Introduction

Natural convection of non–Newtonian fluids occurs in various geophysical and industrial applications including convection of the Earth's mantle, cooking, the construction of oil wells and polymer processing. A range of different non-Newtonian effects occur and our specific interest is in fluids that exhibit resistance to deformation below a critical *yield* stress: so-called visco-plastic or yield stress fluids. Nearly always, yield stress behaviour occurs in combination with other rheological effects. However, in situations where the yielding behaviour is the primary concern or most evident physical behaviour it is common to study such fluids using idealized models, as we do here.

The first and simplest model is the Bingham fluid [1]. Bingham fluids do not deform or flow unless the applied shear stress exceeds the yield stress. Once the fluid is yielded, the strain rate increases linearly with the shear rate and the effective viscosity decreases asymptotically to a high-shear plateau value. The linearity of the flow curve makes the model attractive for analysis of situations where more nonlinear models would not change the qualitative nature of the results.