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The effect of shear thinning and walls on the sedimentation of a sphere in an elastic fluid under orthogonal shear

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ABSTRACT

We investigate the sedimentation of a sphere in a viscoelastic fluid with a cross-shear flow by numerical simulation. The non-Newtonian properties of the suspending fluid determine the settling rate of the sphere. Experiments [Tonmukayakul et al., US Patent Number US8,024,962(B2) (2010); van den Brule and Gheissary, J. Non-Newton. Fluid Mech. 49 (1993) 123–132] have shown the settling rate increases with increase in cross-shear Weissenberg number, *Wi*, in elastic guar gum solutions and decreases in Boger fluids. In the present work, simulations of a sheared viscoelastic flow past a sphere are used to study the effect of the shear-thinning and elasticity of the carrying fluid on the sphere's settling rate. The elastic guar gum solutions are modeled using the Giesekus constitutive model. The parameters are obtained by fitting the rheological data. The drag on the sphere decreases, i.e. the settling rate increases, with an increase in the shear Weissenberg number that is in qualitative agreement with the experiments. The decrease in the drag is primarily due to the decrease in the polymer drag component because of shear-thinning. This is in contrast with the increase in the drag in Boger fluids due to the increase in viscous drag. The effect of different polymer characteristics such as shear thinning and elasticity on the flow field is presented. There is an optimum value for the amount of polymers in the solution for the increase in the viscous drag to overcome the decrease in the polymer drag leading to a net increase in the drag on the sphere. The effect of walls on the drag coefficients in Boger fluids is also investigated. It is demonstrated that the effect of the increase in the drag coefficients with *Wi* is accentuated as the interaction with the wall grows stronger. The wall interactions lead to an increase in viscous shear stresses downstream of the sphere, which causes the increase in the drag.

1. Introduction

The understanding of suspensions of particles in viscoelastic fluids is essential for many engineering applications [1]. One of the applications of interest is in the hydraulic fracturing used during drilling of oil and gas wells [2]. Suspensions of solids in polymeric solutions are pumped during the drilling process to prop open the fracture for enhanced oil recovery. These solids, known as 'proppant', play an essential role in the hydraulic fracturing process. The solids are usually dense mineral particles like sand or sintered bauxite. Sedimentation of these particles reduces the distance travelled in the axial direction of the fracture. The particles need to travel large distances in the axial direction to reach deep into the fracture and increase the oil recovery. Thus, the settling rate of the 'proppant' particles needs to be reduced to increase the effectiveness of the fracturing process. A commonly used polymeric solution to transport the particles is a borate cross-linked guar gum solution [3]. The non-Newtonian properties of these solutions play an important role in their particle-transport capabilities.

In Stokes flow of Newtonian fluids, the settling rate of a spherical particle is not affected by an imposed shear flow in the orthogonal direction to gravity, which is termed a *cross-shear flow* in the paper. But in non-Newtonian fluids, the settling rate is dependent on the shear-thinning (i.e. viscosity reduction with increase of shear rate) as well as the elastic properties of the fluid. There have been both single sphere experiments [4] as well as experiments in more concentrated suspensions [5,6] to determine the settling rate of particles sedimenting in the vorticity direction of an imposed

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shear flow. The single sphere experiments [4] were performed in an aqueous mixture containing polyacrylamide which is a Boger fluid i.e. it is primarily elastic with modest shear-thinning. The concentrated suspension experiments were performed using both Boger fluids [5] as well as borate cross-linked guar gum solutions [6]. The guar gum solutions have both shear-thinning and elastic properties [6]. The settling rate of the sphere is reduced by 50% in Boger fluids due to the cross-shear flow [4]. The reduction in settling rate is more drastic for concentrated suspensions in Boger fluids [5]. But perhaps surprisingly the settling rate increases with the cross-shear flow in elastic guar gum solutions. Thus, the fluid rheology as well as particle concentration are important factors determining the settling rate of the particles under these conditions.

The problem of interest in this paper, namely sedimentation in viscoelastic fluids under imposed shear, has not been fully understood. The experiments for concentrated suspension [5,6] of spherical particles in various viscoelastic liquids have shown that the settling rate can be reduced or increased depending on the rheology of the fluid. The difference between the results of the single sphere experiments [4] and concentrated suspension experiments [5,6] indicate that the particle concentration is very important in determining the settling rates. The problem of single sphere in weakly viscoelastic fluid was investigated by Housiadas and Tanner [7] using perturbation theory which provided some insight about the role of elasticity. Padhy et al. [8] investigated this problem in elastic Boger fluids using numerical simulations. They found the decrease in settling rates observed in the experiments and proposed a mechanism for the effect of elasticity on the settling rates. We study the effect of fluid rheology on settling rate especially the increase in settling rate in elastic guar gum solutions. We also examine the effect of walls in this sedimentation problem as our previous calculations [8] suggested that wall effects played a primary role in explaining the discrepancy between the theory of Housiadas and Tanner [7] and the available experiments [4].

In this context, it is well known that the presence of walls changes the drag on a sedimenting sphere in a Newtonian fluid. The drag correction factor for Newtonian fluids at $Re = 0$ can be obtained using Faxen's law [9]. There has been significant work done to determine the wall effects in elastic fluids under simple sedimentation [10–14]. There is a well known reduction for weak elastic effects in the drag correction factor from the Newtonian value due to the fluid elasticity. But as fluid elasticity is increased, the drag correction factor reaches a plateau and begins to increase. The wall effects on a particle in a viscoelastic shear flow alone have been investigated by Avino et al. [15] using numerical simulations. They studied the dynamics of the particle under imposed confined shear flow. They found particle migration towards the closest wall due to the confinement and viscoelasticity of the fluid. They extended the analysis for time dependent shear flow and obtained many features of previous experimental results [16]. In the present study, we are concerned with the effect of confinement on a sphere sedimenting in a viscoelastic cross-shear flow.

Thus, we study via numerical simulation the problem of a sedimenting sphere with cross-flow in a viscoelastic medium but this time for a broader range of fluid rheology than was considered in our previous work [8]. The simulations are performed for elastic guar gum solutions as well as Boger fluids. The parameters for the simulations are determined by fitting the rheological data measured in experiments [4,6]. The drag coefficients computed from the simulations are directly related to the settling rate of a sphere. The simulation results, indeed, show the increase in settling rate observed in the experiments for elastic guar gum solutions. We analyze these results to determine the effect of shear-thinning and elasticity on the setting rate as discussed in Section 5.1. The effect of walls on the reduction in sedimentation speed in elastic fluids is examined in Section 5.2.

2. Problem formulation

2.1. Problem definition

One can refer to our previous paper [8] for details of the problem set up, but we will review it briefly here. The problem consists of a freely sedimenting sphere moving with constant speed U_∞ under the action of gravity in a cross-shear flow. An inertial frame of reference is attached to the sphere and simulations are performed in this frame of reference. A fluid with uniform velocity of U_i (where $U_i = -U_\infty \tilde{g}_i, \tilde{g}_i$ is a unit vector in the direction of gravity and from Fig. 1, $\tilde{g}_i \equiv \delta_{i1}$) is flowing past a stationary sphere. A no slip boundary condition is applied at the upper and lower plates and these are moved with a speed S in opposite directions. This enforces the cross-shear flow in the problem. The torque on the sphere is constrained to be zero. A rotational velocity, ω_i, is applied to the sphere (only ω_1 is non-zero by symmetry) so that the condition on the torque is satisfied.

2.2. Governing equations

The flow of an incompressible fluid containing polymer additives is governed by the equations of conservation of mass and momentum. The non-dimensionalized forms of these equations are given by:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\beta}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1-\beta}{Re} \frac{\alpha}{Wi} \frac{\partial \tau^P_{ij}}{\partial x_j} \tag{2}$$

where τ^P_{ij} is the stress due to the polymers. The inlet velocity magnitude (U_∞) and the sphere diameter (D) are used to make the variables non-dimensional. The polymer stress and the pressure have been made non-dimensional using μ_p/λ and ρU_∞^2 respectively where λ is the characteristic relaxation timescale of the polymer and μ_p is the polymer contribution to the viscosity. The non-dimensional parameters of interest are the shear Weissenberg number ($Wi = \lambda\dot{\gamma}$), the flow Weissenberg number ($\theta = \lambda U_\infty/D$) and the Reynolds number ($Re = \rho U_\infty D/(\mu_s + \mu_p)$) where $\dot{\gamma}$ is the characteristic shear rate applied across the cell in the $x_2 - x_3$ plane and μ_s is the solvent contribution to the viscosity. The other parameters of note are $\beta = \mu_s/(\mu_s + \mu_p)$ and $\alpha = \dot{\gamma}D/U_\infty = Wi/\theta$.

The two constitutive models used in the paper are the FENE-P model and the single mode Giesekus model [17]. The FENE-P model is used for the constitutive equations of the Boger fluids (Section 5.2). The polymer stress term in Eq. (2) using the FENE-P model is given by [18],

$$\tau^P_{ij} = \frac{c_{ij}}{1 - \frac{c_{kk}}{L^2}} - \delta_{ij} \tag{3}$$

where L is the dimensionless maximum polymer extensibility and c_{ij} is the dimensionless averaged polymer conformation tensor with both made dimensionless using the equilibrium Hookean spring length. The equilibrium Hookean spring length is $(kT/H)^{(1/2)}$ where T is the absolute temperature, k is Boltzmann's constant and H is

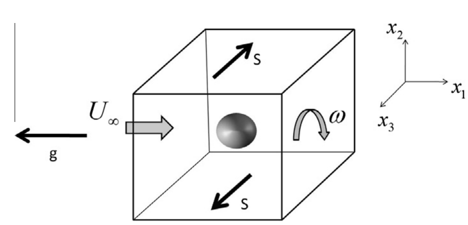

Fig. 1. Schematic of the problem used for simulation.