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Development of a rarefaction wave at discharge initiation in a storage silo—DEM simulations

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ABSTRACT

The generation of a rarefaction wave at the initiation of discharge from a storage silo is a phenomenon of scientific and practical interest. The effect, sometimes termed the dynamic pressure switch, may create dangerous pulsations of the storage structure. Owing to the nonlinearity, discontinuity, and heterogeneity of granular systems, the mechanism of generation and propagation of stress waves is complex and not yet completely understood.

The present study conducted discrete element simulations to model the formation and propagation of a rarefaction wave in a granular material contained in a silo. Modeling was performed for a flat-bottom cylindrical container with diameter of 0.1 or 0.12 m and height of 0.5 m. The effects of the orifice size and the shape of the initial discharging impulse on the shape and extent of the rarefaction wave were examined. Positions, velocities, and forces of particles were recorded every 10^{-5} s and used to infer the location of the front of the rarefaction wave and loads on construction members. Discharge through the entire bottom of the bin generates a plane rarefaction wave that may be followed by a compaction wave, depending on the discharge rate. Discharge through the orifice generates a spherical rarefaction wave that, after reflection from the silo wall, travels up the silo as a sequence of rarefaction–compaction cycles with constant wavelength equal to the silo diameter. During the travel of the wave along the bin height, the wave amplitude increases with the distance traveled. Simulations confirmed earlier findings of laboratory and numerical (finite element method) experiments and a theoretical approach, estimating the speed of the front of the rarefaction wave to range from 70 to 80 m/s and the speed of the tail to range from 20 to 60 m/s.

Introduction

The start of discharge of granular solids from a silo results in a sharp stress redistribution, which can be observed as a ramp of silo pressure. The opening of the discharge gate reduces the vertical pressure and may be associated with an increase in lateral pressure. Zhang, Britton, and Jaremek (1993) reported an increase in lateral pressure exceeding 40% and peaking within 0.7 s of the discharge time for wheat exiting a model silo having a smooth and corrugated wall. Dynamic phenomena during gravity discharge often appear as vibrations (called silo music) or shocks (called silo quakes) (Tejchman & Gudehus, 1993; Wilde et al., 2008). These self-excited effects may result in severe dynamic loads on construction members, such as pressure pulsations and shocks and the non-uniform distribution of pressure and dynamic overpressures. Vibrations and shocks arise from changes in material properties during the flow, such as changes in density, cohesion, degree of mobilization of internal and wall friction (Molenda, Horabik, & Ross, 1995; Muite, Quinn, Sundaresan, & Rao, 2004; Roberts & Wensrich, 2002), and the slip–stick behavior during shear flow (Stasiak, Molenda, Horabik, Mueller, & Opaliński, 2014; Wensrich, 2002).

Under industrial conditions, an initial phase of discharge is frequently accompanied by the propagation of pressure waves from the orifice up through the material (Roberts & Wensrich, 2002). Pressure waves (i.e., compression and rarefaction waves) and associated discontinuities in velocity fields in the granular material during discharge are inherent elements of pulsations and shocks in silo structures (Wensrich, 2002). The phenomena of granular solids leading to the emergence and propagation of stress waves have received growing interest owing to the potential to clarify the state of the granular medium (Hostler & Brennen, 2005) and owing

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to a broad range of industrial applications regarding the dynamic aspects of handling bulk materials (Wang, Wensrich, & Ooi, 2012). In recent decades, the propagation of pressure waves in granular solids has been extensively investigated experimentally (Ben-Dor, Britan, Elperin, Igra, & Jiang, 1997; Börzsönyi & Kovács, 2011; Hostler & Brennen, 2005; Muite et al., 2004; Wensrich, 2002), and theoretically taking an analytical approach (Boutreux, Raphael, & De Gennes, 1997; Cowin & Comfort, 1982; Ocone & Astarita, 1995), and numerical approaches (Herbold & Nesterenko, 2012) based on the finite element method (FEM) (Wang et al., 2012; Wensrich, 2003), and discrete element method (DEM) (Lamei & Mirghasemi, 2011; Melin, 1994; Rong, Negi, & Jofriet, 1995; Wensrich, 2002; Wensrich & Stratton, 2011). Such works have provided insight into mechanisms leading to the emergence, formation, and propagation of the pressure wave.

The mechanism of the creation and propagation of the stress wave in granular solids is not yet completely understood as it involves complicated issues of the nonlinearity and heterogeneity of granular systems. The speed of the wave propagation increases with the overall constraining pressure p. For higher pressures, if the elastic interaction is the Hertz contact law, the speed increases like $p^{1/6}$ (Hostler & Brennen, 2005; Melin, 1994), while the dependence is stronger for low pressures, like $p^{1/4}$, possibly owing to the dominant role of conical asperities in the transition of forces between particles (Goddard, 1990). The dependence of the wave propagation on pressure originates from the barotropy of the material (Wensrich & Stratton, 2011). If the material behind a wave is at a higher pressure than that in front of the wave, the front of the wave tend to break, similar to a breaking wave on a beach (Boltachev, Kaygorodov, & Volkov, 2009; Ocone & Astarita, 1995; Wensrich & Stratton, 2011).

By making constitutive assumptions on the density dependence of the particulate-phase pressure, Ocone & Astarita (1995) showed that rarefaction waves smooth out as they propagate while compression waves reinforce each other to become shocks. Behind the front of a rarefaction wave, material dilates such that the speed of perturbation is lower and the wave profile becomes smoother. Wensrich (2002) showed that a rarefaction wave that is broad and round is immediately followed by a compression wave that is sharper.

When particles start to discharge from a silo, a rarefaction wave is generated by gravity-driven movements of particles losing support from the bottom. The bottom layer of particles starts to accelerate at the beginning of discharge. The contact forces with upper layers weaken and are finally lost. The process moves up the column as the next layers of particles lose their support from particles below. A traveling disturbance of particle velocities, contact forces and distances between particles arises. In terms of continuum mechanics, the rarefaction wave can be treated as a disturbance of the fields of stress, velocity, and density traversing the bedding.

Wensrich (2002) followed the approach of Boutreux et al. (1997) of using the dynamic version of Janssen's equation and applied a hypoplastic model of material to develop the analytical formula for the amplitude of a rarefaction wave. He showed that the amplitude of the rarefaction wave that originates at the discharge orifice grows exponentially as it travels up the silo:

$$\sigma^{+}(h,t)=\sigma_0^{+}e^{h/2H_0}, \quad (1)$$

where $\sigma^{+}(h,t)$ is the stress disturbance in the axial direction at distance h and time t, σ_0^{+} is the initial stress disturbance at the silo discharge orifice, h is the distance traveled by the wave, t is time, $H_0=D/4\mu k$ is the Janssen characteristic depth of the column, D is the diameter of the column, μ is the wall friction coefficient, and k is the lateral-to-vertical pressure ratio.

Table 1
Material and container parameters.

Material:	
Young's modulus, E (MPa)	868
Poisson's ratio, ν	0.18
Restitution coefficient, e	0.5
Particle-wall friction coefficient, $\mu_{\mathrm{p-w}}$	0.3
Particle-particle friction coefficient, $\mu_{\mathrm{p-p}}$	0.482
Particle solid density, ρ (kg/m^3)	1290
Bulk density, ρ_{b} (kg/m^3)	804.8 (793.8)
Void fraction	0.376 (0.385)
Mean particle size, d (mm)	3.79
Particle size range (mm)	3.74–3.84
Particle number	83,000 (118,000)
Time step, Δt (s)	10^{-7}
Container:	
Young's modulus, E (MPa)	2×10^5
Poisson's ratio, ν	0.3
Height, H (m)	0.5
Diameter, D (m)	0.1 (0.12)
Orifice diameter, D_0 (m)	0.036, 0.1 (0.12)
Orifice speed, u_0 (m/s)	0.005, 0.05, ∞

The exponential growth of the amplitude of a rarefaction wave has been confirmed experimentally (Wensrich, 2002) and numerically (Wang et al., 2012; Wensrich, 2002, 2003). As the rarefaction wave dilates the material, there is a decrease in vertical and lateral pressure and a decrease in density directly behind the wave front. The exponential growth of the amplitude of the rarefaction wave obtained from numerical simulations was found to agree well with experimental data (Roberts, 2012; Roberts & Wensrich, 2002).

A wide range of measurements of the wave speed in granular beds has been reported in the literature; i.e., from 50 to 500 m/s for glass and sand (Ben-Dor et al., 1997; Börzsönyi & Kovács, 2011; Hostler & Brennen, 2005; Wensrich, 2002). A probable reason for the broad range of wave speeds given by the authors is a difference in the bulk properties of materials used in experiments and modeling parameters; i.e., the initial bulk density, degree of nonlinearity of the force–displacement relationship for particle–particle interaction, and the stress–strain relationship, continuum mechanics equivalent. Additionally, different frictional, viscous or plastic damping and different boundary conditions and construction parameters of the bin affect the wave speed. The stress history may be another factor in play.

The purpose of the present study was to examine the formation and propagation of a rarefaction wave in a granular material filling a flat-bottomed cylindrical model silo employing the DEM, and to compare findings with those of experiments and finite element modeling. Some, not yet clearly explained, issues are addressed; i.e., the effects of the orifice size and the shape of the initial discharge impulse on the shape and extent of a rarefaction wave.

DEM modeling

A series of DEM simulations was performed with an assembly of 83,000 (and 118,000) spherical particles with a random uniform distribution of diameters of 3.79 ± 0.05 mm. The Hertz–Mindlin theory of elastic frictional collisions between particles (and between particles and a wall) was used for simulations following the work of Mindlin and Deresiewicz (e.g., Di Renzo & Di Maio, 2004). A damping term in the contact model was adopted as proposed by Tsuji, Tanaka, and Ishida (1992); i.e., it was assumed that the coefficient of restitution has a constant value for given particle properties. The material parameters of particles were taken as those measured for wheat kernels by Wiącek, 2008; i.e., Young's modulus E = 868 MPa, Poisson's ratio ν = 0.18, coefficient of restitution e = 0.5, and coefficient of particle–wall friction $\mu_{\mathrm{p-w}}$ = 0.3 (Table 1). Numerical experiments were carried out for two flat-bottomed