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# When is a parameterized controller suitable for adaptive control?

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ABSTRACT

In this paper we investigate when a parameterized controller, designed for a plant depending on unknown parameters, admits a realization which is *independent* of these parameters. It is argued that adaptation is unnecessary for this class of parameterized controllers. We prove that standard model reference controllers (state and output-feedback) for linear time invariant systems with a suitably chosen filter at the plant input admit a parameter independent realization.


## 1. Problem formulation

The following question is addressed in this paper. Consider a linear time-invariant (LTI) parameterized plant:

$$\begin{aligned}\dot{x} &= A(\theta)x + B(\theta)u\\ y &= C(\theta)x, \end{aligned} \tag{1}$$

with state $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ the plant input and output, respectively, and $\theta \in \mathbb{R}^q$ a vector of constant parameters; and an LTI parameterized controller

$$\begin{aligned}\dot{\xi} &= A_c(\theta)\xi + B_c(\theta)\begin{bmatrix} y \\ r \end{bmatrix}\\ u &= C_c(\theta)\xi, \end{aligned} \tag{2}$$

where $\xi \in \mathbb{R}^{n_\xi}$ and $r \in \mathbb{R}^s$ is some external reference. Under which conditions does there exists a change of coordinates

$$\begin{bmatrix} x \\ \chi \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ T_1(\theta) & T_2(\theta) \end{bmatrix}\begin{bmatrix} x \\ \xi \end{bmatrix}, \tag{3}$$

with $T_1(\theta) \in \mathbb{R}^{n_\xi \times n}$ and $T_2(\theta) \in \mathbb{R}^{n_\xi \times n_\xi}$, with

$$\det T_2(\theta) \neq 0$$

such that the dynamics of the controller in the coordinates $\chi$ takes the form

$$\dot{\chi} = A_N\chi + B_N\begin{bmatrix} y \\ r \end{bmatrix} u = C_N\chi + D_N y, \tag{4}$$

with the matrices $A_N$, $B_N$, $C_N$ and $D_N$ *independent* of the parameters $\theta$. A controller verifying these conditions is said to admit a parameter-independent realization or, in short, that it is a PIRC.

Our interest in this question stems from model reference control (MRC) where, as indicated by the name, the control objective is that the closed-loop mapping $r \mapsto y$ matches a desired reference model [4]. In its adaptive version—*i.e.*, MRAC—it is assumed that the parameters of the plant $\theta$ are unknown but a parameterized controller that matches the reference model is assumed to be known. In this case, the reference model is, naturally, independent of the plant parameters. The scheme is made adaptive replacing in the controller the unknown vector $\theta$ by an on-line estimate $\hat{\theta}$, which is generated via a parameter identifier. The rationale behind this approach is, clearly, that if the estimated parameters $\hat{\theta}$ converge to their true value $\theta$ then —modulo some technical conditions—the adaptive controller will achieve the control objective.[1]

From this perspective it is clear that for a PIRC there is no need to make the scheme adaptive! Indeed, we can simply plug-in the LTI parameter-independent controller (4) that will generate the same control input for the plant and, due to the invariance to coordinate transformations of the transfer matrix, it will match the desired reference model. Surprisingly, this very simple observation has been totally overlooked by the adaptive control community.[2] As discussed in Section 4 this kind of analysis puts a serious question

[1] As is well-known [13], consistent parameter estimation is not necessary to achieve the control objective, this is the so-called self-tuning property of adaptive control.

[2] In [7] a somehow related question that arises in identification theory is addressed.

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mark on the usefulness of some control schemes recently reported in the control literature.

The main contributions of the paper are the proofs of the following facts.

R1 Standard state-feedback and output-feedback MRC are *not PIRC*.
R2 Adding *any* LTI, strictly proper, input filter to the standard state-feedback MRC makes it a PIRC.
R3 Output-feedback MRC becomes a PIRC adding an LTI input filter of *relative degree* not smaller than the relative degree of the plant.

The first result is rather obvious and, as seen below, the proof is straightforward. In [9] R2 was established for the case of first-order plants with first order filters and a regulation objective, *i.e.*, $r$ constant. The generalization to $n$-th order plants was reported in [10] for the case of stabilization, *i.e.*, $r=0$. Related developments have been reported in [6]. A modified version of output-feedback MRC reported in [3], which includes an input filter, was shown to be an LTI controller in [2]. To the best of our knowledge, the case of (classical) output-feedback MRC is addressed for the first time in this paper.

The remaining of the paper is organized as follows. State-feedback MRC, and its filtered version, are studied in Section 2. Section 3 is devoted to output-feedback MRC. The analysis of state-feedback MRC is carried-out using state realizations of both, the plant and the controller. On the other hand, for the analysis of output-feedback MRC it is more natural to use polynomial representations. We wrap-up the paper with some concluding remarks regarding the adaptive implementations of the various MRC in Section 4. An abridged version of this paper, without the proofs of the propositions, has been reported in [11].

## 2. State-feedback model reference control

In its simplest version state-feedback MRC deals with single-input, LTI systems of the form

$$\dot{x} = Ax + bu \tag{5}$$

where $x \in \mathbb{R}^n$ is assumed to be *measurable*,

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & -a_3 & \cdots & -a_n \end{bmatrix}$$

where $a_i \in \mathbb{R}, i \in \overline{n} := \{1, \ldots, n\}$ are unknown coefficients, and $b = e_n$— the $n$-th vector of the Euclidean basis.[3] We are also given a reference model

$$\dot{x}_m = A_m x_m + br$$

where the state $x_m \in \mathbb{R}^n$ and $r \in \mathbb{R}$ is a bounded reference, $A_m \in \mathbb{R}^{n \times n}$ is the Hurwitz matrix

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1^m & -a_2^m & -a_3^m & \cdots & -a_n^m \end{bmatrix}$$

with $a_i^m \in \mathbb{R}_+$, $i \in \overline{n}$, designer chosen coefficients.

[3] This assumption is made to simplify the notation and without loss of generality. See Remark R3 in [12].

Defining the vector of *unknown* parameters

$$\theta = \mathrm{col}(a_1 - a_1^m, a_2 - a_2^m, \ldots, a_n - a_n^m),$$

where $\mathrm{col}(\cdot)$ denotes column vector, it is clear that

$$A + b\theta^\top = A_m. \tag{6}$$

Hence, invoking (6), we can write (5) in the equivalent parameterized system form

$$\dot{x} = (A_m - b\theta^\top)x + bu. \tag{7}$$

A parameterized controller that achieves the model matching objective is clearly

$$u = \theta^\top x + r. \tag{8}$$

The error dynamics takes the form

$$\dot{e} = A_m e \tag{9}$$

where $e := x - x_m$.

This MRC is made adaptive adding an identifier that generates the estimated parameters, denoted $\hat{\theta} \in \mathbb{R}^n$.

An alternative approach, pursued *e.g.* in [3], consists in *addition of a filter* at the plant input, *i.e.*, in computing $u$ via

$$u = F(p)(\hat{\theta}^\top x + r)$$

where $p := d/dt$ and $F(p) \in \mathbb{R}(p)$ is strictly proper and stable. More precisely,

$$F(p) = \frac{N_f(p)}{D_f(p)}, \tag{10}$$

where

$$D_f(p) = \sum_{i=0}^{n_{D_f}} d_{fi} p^i, \quad N_f(p) = \sum_{i=0}^{n_{N_f}} n_{fi} p^i$$

with $n_{D_f} > n_{N_f}$ and $D_f(p)$ and $N_f(p)$ are coprime with designer chosen coefficients.

A state realization of the filtered state-feedback MRC is

$$\dot{\xi} = A_f \xi + b_f(\theta^\top x + r)$$
$$u = c_f^\top \xi, \tag{11}$$

where

$$F(p) = c_f^\top (pI - A_f)^{-1} b_f,$$

and $n_\xi = n_{D_f}$.

It should be underscored that the addition of such a filter is of questionable interest. On one hand, the original objective of model matching is now unachievable, *i.e.*, the plants state $x$ is unable to follow the state of the reference model $x_m$ for all references $r$. On the other hand, as clearly shown in [5] the inclusion of the filter systematically degrades the performance of its adaptive implementation.

**Proposition 1.** *Consider the plant (7).*

(i) *The classical state-feedback MRC (8) is not a PIRC.*
(ii) *For any strictly proper filter (10), the filtered state-feedback MRC (11) is a PIRC.*

**Proof.** We will prove first (ii). Towards this end we will show that applying to (11) the partial coordinate transformation (3), with[4]

$$T_1 = b_f b^\top$$
$$T_2 = I_{n_\xi},$$

[4] Notice that the proposed coordinate transformation is *independent* of $\theta$.