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On eigenstructure assignment using block poles placement

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ABSTRACT

In this paper a design process is described to achieve eigenstructure assignment using block poles. Systems described in state space equations are transformed to systems in matrix fractions description and for the latter's, eigenvalues are called latent values and eigenvectors are called latent vectors. The method proposed here allows assigning, exactly, the whole set (and even more) of latent values and vectors obtained from a desired eigenstructure using dynamic compensation. The necessary condition is the block-controllablity or block-observability of the system.

1. Introduction

Let a Multiple Input Multiple Output (MIMO) system be in standard state space description (SSD) which is defined by the following dynamical state equations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

where $A \in \mathcal{R}^{n \times n}$, $B \in \mathcal{R}^{n \times m}$ and $C \in \mathcal{R}^{p \times n}$.

For more convenience we suppose that the input–output matrix is null.

The eigenstructure of this system consists in the set of n eigenvalues and its corresponding eigenvectors of the state matrix A:

$$Av_i = \lambda_i v_i \quad i = 1:n \tag{2}$$

$$w_j A = \lambda_j w_j \quad j = 1:n \tag{3}$$

where λ_i is the ith eigenvalue of A, $v_i \in \mathcal{R}^{n \times 1}$ is the associated right eigenvector and $w_j \in \mathcal{R}^{1 \times n}$ is the associated left eigenvector.

Eigenstructure assignment (EA) consists in determining a feedback gain matrix (state, output) in order to assign a desired eigenstructure to the closed loop system thus obtained which verifies in general some specific requirements.

The problem of eigenstructure assignment is of great importance in control theory and applications because the stability and dynamic behavior of a linear multivariable system are governed by the eigenstructure of the system [16].

Eigenstructure assignment is a natural choice for the design of any control system whose desired performance is readily represented in terms of an ideal eigenstructure [25].

The problem of eigenstructure assignment (and poles placement) has a long history going back to [20]. And it has attracted much research interest over many years; see for instance the survey paper [28].

The degrees of freedom available in eigenstructure assignment using state-feedback control are well known. If we consider the linear time-invariant system described by the state equation (1) and applying linear state feedback $u = Kx$ with $K \in \mathcal{R}^{m \times n}$, then it was shown by Wonham, [30], that a matrix K can be found to an arbitrary self-conjugate set of eigenvalues if the system is controllable. In 1976, Moore [18] described the freedom available to assign eigenvectors for an arbitrary self-conjugate set of eigenvalues using state feedback. He gives both necessary and sufficient conditions for a full-state feedback matrix K to exist. Furthermore, if such a feedback matrix K exists and B is of full rank then K is unique.

If B is full rank, a maximum of n eigenvectors can be partially assigned with a minimum of m entries in each eigenvector arbitrarily chosen. For a particular problem, it may be desirable to consider more than the minimum number of entries in a given eigenvector. In this case, the set of basis vectors which span the allowable subspace must be determined and a best possible achievable eigenvector chosen [10].

The authors of the interesting paper [2] gave the importance of the eigenstructure assignment in control linear systems with an extensive flight control example. They discussed the conditions for

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the number of reachable eigenvectors, and presented the different techniques to assign them, via full state feedback, output feedback and constrained output feedback.

In more recent papers, [32] proposed an iterative algorithm, based on alternating projections ideas. Given n subsets of the complex plane, the algorithm is used to find a static output feedback that places a pole for each subset. Kimura' condition ($m+p>n$) is generally considered as the best sufficient condition for a problem of EA to always have a solution, and Wang's condition ($mp>n$) is less restrictive but the solution if it exists is not easily obtained. [3] proposed a simple non-iterative technique, based on eigenstructure assignment, which places, by static output feedback, $m+p$ poles when $mp>m+p$. So if $m+p=n$ and $mp>n$ then the method assigns the whole desired spectrum [4].

In [7] a two-stage design process is used to formulate the gain matrix. In the first stage a subset of desired eigenvectors is assigned, then, in the second stage, a dual set is assigned if necessary and sufficient conditions are met. In [19] an algorithm is presented which exploits unused design freedom to introduce structure to the resulting gain matrix without affecting the assigned eigenstructure by output feedback.

Hippe and O'Reilly [11] proposed a solution to the problem of parametric eigenvalue assignment using general dynamic output feedback compensator of order l ($0 \le l \le n$). they showed that $n+l$ eigenvalues are assignable by a dynamical compensator of order $l=\nu-1$ where ν is either the system controllablity index or the system observability index. And Magni [17] proposed a control design approach based on EA by dynamic feedback to handle simultaneously robustness against real parameter variations, and the use of structured gain including scheduled gains.

Analysis and synthesis based on the matrix fraction description (MFD) of multivariable systems have received a great deal of attention for the past few decades [5,6,12,29]. The famous book by Kailath [12] documents the aspect of system theory in Polynomial Matrix fraction thoroughly. And in Stefanidis et al. [26] the numerical aspect of polynomial matrices and their use in Control theory (closed loop system compensator design) has been detailed and illustrated with many design examples.

In [21] a direct pole placement algorithm is introduced for dynamical systems having a block companion matrix A. The algorithm utilizes well established properties of matrix polynomials. Pole placement is achieved by appropriately assigning coefficient matrices of the corresponding matrix polynomial then computing the state feedback allowing the placement.

A large-scale MIMO system described by state equations in general coordinates is often decomposed into small subsystems, from which the analysis and design of the MIMO system can be easily performed. In [23] a new block-pole placement for the state-feedback block decomposition of a class of MIMO systems is derived.

In [1] a matrix fraction description for the development of a compensator for a linear system using a state feedback law and estimator design for that system is presented and in [27] a new approach to compute the coprime MFD and the state feedback gains of MIMO systems is presented.

In this paper we propose a new method based on MFD and matrix polynomials to achieve eigenstructure assignment by block poles placement using a dynamic compensator. For a system described in SSD, the first step consists in converting it to MFD, then convert the desired eigenstructure into a latent structure, and construct desired block poles. A desired denominator of the closed loop system is thus obtained and the compensator to achieve this placement is computed by solving a Diophantine equation.

So for an m-input, p-output, n-state system with a controllability index (or observability index) equal ν, the proposed method can place ν block poles of dimension $m\times m$ (or $p\times p$), thus assigning n eigenvalues and latent vectors (right or left) using a static compensator. But more important, we can determine a dynamic compensator of degree $l\ge 1$ by placing a number $\nu+l$ block poles, thus assigning a number $n+l{*}m$ (or $n+l{*}p$) of eigenvalues and its corresponding latent vectors, as long as the following conditions are verified: (i) the system is block controllable (or block observable), (ii) there exist in a set of $n+l{*}m$ (or $n+l{*}p$) latent vectors, ν subsets which are linearly independent, (iii) the conditions for the resolution of the obtained Diophantine equation are met, (iv) and finally the computed compensator is proper and stable. If the computed compensator is not proper or stable, its degree is increased by increasing the number of the desired eigenvalues and its corresponding latent vectors.

However, the issue of minimal compensator degree is less important, in that increased degree provides additional useful degrees of design freedom beyond arbitrary eigenvalue assignment [11].

The design process offers a larger degree of freedom (more than the set of original desired eigenvalues can be assigned) and the algorithm is direct and allows assigning desired eigenstructure through block poles but block zeros can also be placed simultaneously.

Compared to the previous works and to the best of the authors' knowledge nobody considered using block poles placement for systems described by matrix fractions to assign an eigenstructure using dynamic compensators.

The paper is organized as follows: first a section of small recalls on polynomial matrices, MFD, and EA. Then the second section will include the first steps to perform eigenstructure assignment: transformation from SSD to MFD, desired block poles construction, desired closed loop denominator construction. It is followed with the section which deals with the dynamic compensator design. An illustrative example is given all along the design steps. The emphasis is given on the usage of the design process rather than on a rigorous theorem-proof scheme. Finally comments and a conclusion will finish the paper.

2. Theoretical preliminaries

The details of this section are generally well known but are repeated for completeness.

2.1. Block companion forms

Let a system be described by standard state equations as in Eq. (1) supposed controllable and/or observable. In this case the system can be transformed into companion forms (observer, controller, etc.) through similarity transformations.

Two transformations are needed in our case: block controller form and block observer form.

2.1.1. Conditions

To convert a system described in SSD as in Eq. (1), the system must satisfy the following conditions [26]:

- the number of states is a multiple of the number of inputs or outputs: $\nu=n/m$ or $\nu=n/p$ (νbeing an integer).
- the system is controllable or observable.

In this case, the system is said block-controllable or block-observable.

Remark 1. If the dimension of a system matrix is not an integer multiple of the number of inputs or outputs some nondominant eigenvalues can be added and placed at the diagonal entries of the system matrix to enlarge the dimension [22].