European Journal of Control (2012)4:348–355

DOI:10.3166/EJC.18.348–355

**European Journal of Control**

# Trajectory Tracking for a Wheeled Mobile Robot Using a Vision Based Positioning System and an Attitude Observer

Martin Velasco-Villa**, Eduardo Aranda-Bricaire***, Hugo Rodríguez-Cortés*, Jaime González-Sierra****

CINVESTAV-IPN, Departamento de Ingeniería Eléctrica, Sección de Mecatrónica, Av. IPN 2508, Col. San Pedro Zacatenco,
CP 07360, México D.F., México

*The trajectory tracking problem for a wheeled mobile robot is addressed and solved by means of a partial state feedback strategy based on measurements from an indoor vision based absolute positioning system. The Cartesian coordinates provided by the localization system are fed to the proposed observer in order to estimate the orientation of the vehicle. It is shown that the combination of a classical dynamic full information controller with an asymptotically convergent vehicle attitude observer, designed using the immersion and invariance technique, yields a locally asymptotically stable closed-loop system. Real time experiments show the performance of the proposed control scheme.*

**Keywords:** Mobile robot, nonlinear observer, vision based positioning system, trajectory tracking

## 1. Introduction

Wheeled mobile robots are devices able to move on a workspace with different degrees of autonomy. The simplest example of a wheeled mobile robot is provided by the so-called unicycle, shown in Fig. 1. The Cartesian coordinates and the attitude of an unicycle-type mobile robot with respect to an inertial frame are denoted by $(x_1, x_2)$ and $\theta$, respectively. Despite the apparent simplicity of this device, controlling it yields to different kinds of challenges [6]. Notice first that system in Fig. 1 is underactuated and satisfies nonholonomic constraints, i.e., the velocities of the system satisfy non integrable constraints. This fact renders the control problem for this class of systems an interesting issue. As stated in [5] such nonholonomic systems cannot be stabilized by continuously differentiable, time invariant, state feedback controllers. For this reason the stabilization problem of underactuated nonholonomic systems has attracted the interest of many researchers [12], who had proposed time varying controllers, discontinuous controllers and composite strategies.

From a technological point of view some other problems arise. For instance, the estimation of the position and attitude of a mobile robot is not a simple task [7, 8]. Different approaches have been proposed to solve the estimation problem over the past decade and there is a vast literature, see for instance [2, 4, 13, 15]. Amongst the absolute positioning methods, video cameras and ultrasonic emitter-receiver arrangements are frequently encountered. Concerning relative positioning methods, dead reckoning is widely used because of its simplicity; moreover it gives an attitude estimation, but it is unsuitable for long distances due to errors associated mainly with noise and slipping conditions [3].

In this paper, it is shown that the fusion of a full information variable structure dynamic state feedback

---

*Correspondence to: H. Rodríguez-Cortés, E-mail: hrodriguez@cinvestav.mx
** E-mail: velasco@cinvestav.mx
*** E-mail: earanda@cinvestav.mx
**** E-mail: jamesgsjr@hotmail.com

*Received 4 June 2010; Accepted 25 August 2011*
*Recommended by A. Astolfi, A.J. van der Schaft*

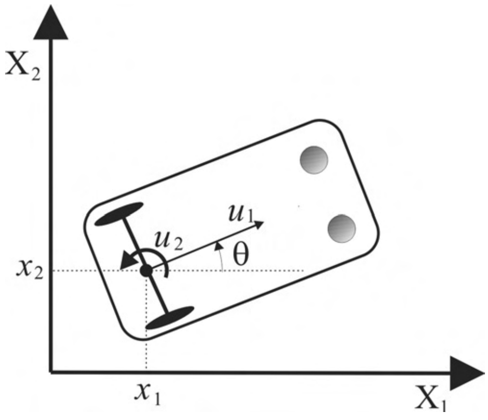

**Fig. 1.** Unicycle mobile robot.

controller with a globally asymptotically stable attitude estimator yields a locally asymptotically stable solution to the trajectory tracking problem for the (2,0) wheeled mobile robot. The variable structure control scheme allows to overcome the singularity that arises when the robot's longitudinal velocity is zero. The proposed attitude estimator, based on the immersion and invariance technique, requires information about the robot's position. In turn, this information is obtained by means of an indoor absolute localization system based on visual servoing. The proposed control strategy is evaluated experimentally on a laboratory prototype.

The paper is organized as follows. In Section 2, the wheeled mobile robot considered in this paper and the full information dynamic state feedback controller that solves the trajectory tracking problem are presented. Section 3 contains the main contributions of this paper, that is, the attitude estimator design and the stability analysis of the partial information dynamic state feedback in closed-loop with the mobile robot dynamics. Section 4 describes the experimental setup as well as the methodology employed to estimate the position of the robot on the plane. Another contribution of this paper, namely the experimental evaluation of the proposed control strategy, is presented in Section 5. Finally, in Section 6 some conclusions are given.

## 2. Class of Systems and Dynamic State Feedback Control

The kinematic model of the unicycle mobile robot shown in Fig. 1 is described by the following equations,

$$\begin{aligned} \dot{x}_1 &= u_1 \cos(\theta) \\ \dot{x}_2 &= u_1 \sin(\theta) \\ \dot{\theta} &= u_2, \end{aligned} \tag{1}$$

where $u_1$ is the unicycle velocity perpendicular to the wheels axis and $u_2$ is the yaw rate, [6]. The trajectory tracking problem is solved through a variable structure control strategy based on full dynamic feedback linearization. This control strategy was already proposed in [9, 10] to solve the trajectory tracking problem for the kinematics model of an articulated mobile robot with $n$ trailers.

In order to give a solution to the trajectory tracking problem, consider the extended kinematics model of the unicycle mobile robot given by the following equations,

$$\begin{aligned} \dot{x}_1 &= u_1 \cos(\theta) \\ \dot{x}_2 &= u_1 \sin(\theta) \\ \dot{\theta} &= u_2 \\ \dot{u}_1 &= v_1 \end{aligned} \tag{2}$$

where $v_1$ is a new control input. It is well known [6] that system (2) with the output

$$y_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

is input-output linearizable by static state feedback and possesses trivial zero dynamics, that is, system (2) with output $y_1$ has a vector relative degree $[2,2]^\top$ with decoupling matrix defined as

$$M_1(\theta, u_1) = \begin{bmatrix} \cos(\theta) & -u_1 \sin(\theta) \\ \sin(\theta) & u_1 \cos(\theta) \end{bmatrix}. \tag{3}$$

Consequently, the dynamic feedback

$$U_1 = \begin{bmatrix} v_1 \\ u_2 \end{bmatrix} = M_1^{-1}(\theta, u_1) \begin{bmatrix} \ddot{y}_{11d} - k_{d_{11}} \dot{y}_{11e} - k_{p_{11}} y_{11e} \\ \ddot{y}_{12d} - k_{d_{12}} \dot{y}_{12e} - k_{p_{12}} y_{12e} \end{bmatrix}, \tag{4}$$

where

$$y_{11e} = x_1 - y_{11d}, \quad y_{12e} = x_2 - y_{12d}$$

with $y_{11d}$ and $y_{12d}$ the desired trajectory, $k_{d_{1i}}$ and $k_{p_{1i}}$, $i = 1, 2$, positive constants with the property that polynomials $s^2 + k_{d_{1i}} s + k_{p_{1i}}$ have all roots with negative real part, is such that

$$\begin{aligned} &\lim_{t\to\infty} y_{11e} = 0, \quad \lim_{t\to\infty} \dot{y}_{11e} = 0, \\ &\lim_{t\to\infty} y_{12e} = 0, \quad \lim_{t\to\infty} \dot{y}_{12e} = 0 \end{aligned} \tag{5}$$

exponentially outside the manifold

$$\mathcal{S}_1 = \{(x_1, x_2, \theta, u_1) \in R^4 \mid u_1 = 0\}. \tag{6}$$

As should be evident from (3) the state feedback (4) cannot follow trajectories that drive the longitudinal velocity