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Transfer Matrices and Advanced Statistical Analysis of Digital Controlled Continuous-Time Periodic Processes with Delay

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The paper investigates the response of a closed-loop sampled-data system $\mathcal{S}$ with an finite dimensional linear continuous periodic (FDLCP) process and pure delay to a stationary stochastic input. Since the variance of the quasi-stationary output of the system $\mathcal{S}$ is a periodic function of the time, an advanced form of statistical analysis is required. The paper introduces the advanced statistical analysis problem, and its solution is derived on basis of the parametric transfer matrix (PTM) concept. Moreover, a characteristic polynomial is defined, which gives necessary and sufficient conditions for the asymptotic stability of the system $\mathcal{S}$. An example demonstrates the advanced statistical analysis procedure.

Keywords: Periodic control systems, sampled-data systems, continuous-time process, stochastic signals, statistical analysis.

1. Introduction

One of the actual problems in modern control theory consists in controlling linear periodic (LP) systems of several types. Most investigations for this class of systems are devoted to finite dimensional linear continuous periodic (FDLCP) systems and linear sampled-data (LSD) systems containing continuous linear time-invariant (LTI) elements. Various aspects for this class of problems could be find in the broadly based literature, e.g., [2]–[33], [36]–[44] and the references therein. The analysis of these contributions shows that, as in the classical theory of linear time-invariant (LTI) systems, for the mathematical description of LP processes and the solution of control problems for such processes, at present two approaches have been established—time and frequency domain approaches. One actual approach in time domain consists in applying the theory of state space equations and the lifting technique. The actual state of this method is described in the monographs [7, 10]. The frequency approach at the moment develops in two parallel branches. The first branch is based on the application of infinite dimensional matrices and determinants [15], [25], [30], [36, 37], [42–44]. The second approach does not apply infinite matrices, but it is sustaining on the parametric transfer matrix (PTM) concept $W(s,t) = W(s,t+T)$, where T is the period of the system. In addition to the complex variable s, the PTM $W(s,t)$ depends on the time t as a parameter. The mathematical foundation of the *PTM* theory for FDLCP systems and the technique for its application is argued in [18, 19, 27], and analogue questions for LSD systems have been investigated in [28, 29].

One of the fundamental problems in the theory of LP Systems consists in their behavior, when stationary stochastic signals act on the input. The statement and solution of this problem for LP systems are essentially different from the analogue problem for LTI systems. The reason for that the stationary response of an asymptotically stable

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LP system with period T to a stationary stochastic input is a periodically non-stationary stochastic process, an its variance $d(t)$ is a periodic function with period T, i.e., $d(t) = d(t+T)$. As examples show [23, 28], the variation of the variance inside the period can be considerable. Therefore, the value of the mean variance

$$\overline{d} = \frac{1}{T}\int_0^T d(t)\,\mathrm{d}t$$

alone does not give an adequate information about the correct operation of a system in the quasi-stationary mode. This statement is also valid for the $\mathcal{H}_2$ norm $\|\mathcal{S}\|_2$ of the system $\mathcal{S}$, which is connected with the mean variance by the relation

$$\|\mathcal{S}\|_2 = \sqrt{\overline{d}}.$$

It follows from the above said that the mean variance or $\mathcal{H}_2$ norm alone, in the general case do not guarantee the precision demands on the system. Therefore, during the design of LP systems, the dependence of the variance $d(t)$ over the complete period has to be studied. The calculation of the $\mathcal{H}_2$ norm alone is not sufficient. In the paper below, the calculation problem for the variance $d(t)$ is called the advanced statistical analysis (ASA) problem.

An appropriate instrument for the solution of the ASA problem for several classes of LP systems is the PTM method. The key consists in the integral formula

$$d(t) = \frac{1}{2\pi \mathrm{j}}\int_{-\mathrm{j}\infty}^{\mathrm{j}\infty} \mathrm{trace}\left[W'(-s,t)W(s,t)\right]\,\mathrm{d}s.$$

In [18, 19, 27], this formula was applied to FDLCP systems. Analogue results for LSD systems can be found in [28, 29]. For practical applications the above integral formula can be divided into several partial problem:

a) Construct the PTM for an LTP system of given type.
b) Study the properties of the PTM $W(s,t)$ as a function of the arguments s and t.
c) Prepare computation methods for the corresponding integrals.

For FDLCP and LSD systems, these programs have been realised in [19, 23, 28, 29].

An essential generalization of the PTM method is its extension to LSD systems including FDLCP elements. This system class is of exceeding practical and theoretical interest. However, this problem is studied occasionally, and only few papers have been published. In particular, the papers [18, 21, 22] consider the stabilization of such systems. In [23] the ASA problem of a single-loop

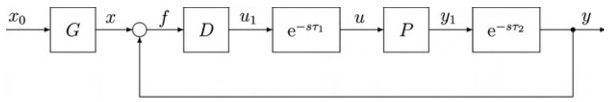

Fig. 1. Digital controlled periodic process with delay.

sampled-data system with an FDLCP process is constructed. The present paper extends the method in [23] to the case, where the system has additional pure delays at the input or output of the digital controller. This problem formulation is more realistic and allows to take into account the computational delay, which always happens in sampled-data systems.

2. System Description and Problem

The paper considers the system $\mathcal{S}$ with the structure shown in Fig. 1. In Fig. 1 the small letters denote vector signals with the following dimensions: $x_0 - l \times 1, x, f, y_1, y - n \times 1, u, u_1 - m \times 1$. Furthermore, G is an *LTI* filter with the strictly proper transfer matrix $G(s)$ of size $n \times l$, where all its poles are located in the open left halfplane. Moreover, τ_1 and τ_2 are nonnegative constants, characterizing the pure delay in different parts of the control loop.

The block P in Fig. 1 denotes the FDLCP process, which is described by the state equation

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = A(t)v(t) + B(t)u(t) \tag{1}$$

and the output equation

$$y_1(t) = C(t)v(t), \tag{2}$$

where $v(t)$ is the $p \times 1$ state vector, and $A(t) = A(t+T)$, $B(t) = B(t+T)$, $C(t) = C(t+T)$ are continuous matrices of the dimensions $p \times p$, $p \times m$, and $n \times p$, respectively.

Denote by $H(t)$ the $p \times p$ Cauchy matrix of equation (1), satisfying the matrix equation

$$\frac{\mathrm{d}H(t)}{\mathrm{d}t} = A(t)H(t), \quad H(0) = I_p,$$

where I_p is the $p \times p$ identity matrix. Traditionally, see e.g., [38], the matrix

$$M = H(T)$$

is called the monodromy matrix of equation (1). As known [38], for any integer k

$$H(t+kT) = H(t)M^k, \quad H^{-1}(t+kT) = M^{-k}H^{-1}(t).$$

In particular, we have

$$H^{-1}(t-T) = MH^{-1}(t). \tag{3}$$