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Event-based controller synthesis by bounding methods ☆

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ABSTRACT

Two event-triggered algorithms for digital implementation of a continuous-time stabilizing controller are proposed in this work. The first algorithm updates the control value in order to keep the time evolution of a given Lyapunov-like function framed between two auxiliary functions; whereas the second one actualizes the control value so that the state trajectory of the system stays enclosed between two a priori defined templates. In both cases, a natural hybrid formulation of the event-based stabilizing control problem is used to prove the main results of this work. Furthermore, the existence of a minimum inter-event time greater than zero is proved. Numerical simulations are provided to illustrate the digital implementation of the event-sampling algorithms for nonlinear systems.

1. Introduction

Usually, state feedback control laws applied to dynamical systems are implemented digitally; and the core idea of this discrete-time implementation consists in sampling the continuous-time control law periodically with a sufficiently small sampling period. However, this procedure may be constrained in practice. On the one hand, reducing the sampling period to a level that preserves acceptable performance of the controlled system requires a fairly powerful and so expensive hardware [8]. On the other hand, today's systems are complex and compound by several subsystems controlled by a single CPU. Consequently, reducing the communication between the CPU and the subsystems is a challenge of great interest which allows enhancing the ability to control more complex systems and reducing energy consumption.

To reach this goal, numerous control strategies called event-based approaches have been proposed in the literature, see [14] for a recent framework encompassing the most recent existing event-triggered control techniques. They aim to update the control value only when a significant event occurs. Usually, this event is defined as a deviation threshold on the state vector or on the input vector. In this work, new criteria to design event-triggered sampling algorithms for a large class of nonlinear systems are proposed where the control updating decision is based on the dynamical behavior of auxiliary systems.

The first sampling algorithm updates the control value in order to guarantee that the Lyapunov-like function of the event-based system stays framed at each time instant between the Lyapunov functions of the auxiliary systems. The global stability of the event-controlled system is guaranteed without requiring the ISS stability of each subsystem and satisfying a supplementary small gain condition as needed in [3], where scalar interconnected systems are considered. The second sampling algorithm is based on a component by component comparison of the plant state with a priori defined state templates. In fact, in this case, the control updating procedure aims to force the state trajectory of the event-based system to never leave the state enclosure generated by the auxiliary systems. Moreover, the existence problem of a minimal inter-event time bigger than zero is solved. This algorithm is inspired from the design of event-based controllers by using dead-band methods (see, e.g., [9] for an introduction of this method). Consider in particular [13] where only single-input-single-output linear systems are considered. See also recent papers on send-on-delta control techniques dealing with bandlimited signal as in [2].

A preliminary version of this work focused on the case of linear systems has been presented in [11].

The paper is organized as follows. In Section 2 preliminary definitions and notions about hybrid systems, useful to prove our main contributions, are introduced. The problem under consideration is formulated in Section 3 as stability issue of hybrid systems. Sections 4 and 5 state the main contributions of this work regarding the design of event-triggered state feedback controls for nonlinear systems. Numerical simulations are provided in Section 6 when

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focusing on a nonlinear system borrowed from [3]. Section 7 collects concluding remarks.

Notation: In this paper the Euclidean inner product of two vectors x and y will be denoted by $x \cdot y$, the induced norm will be denoted by $|\cdot|$. Given a set $\mathcal{A}$, and a point x, $|x|_{\mathcal{A}}$ is the distance of x relative to $\mathcal{A}$, that is $\inf_{z \in \mathcal{A}} |x-z|$. int $\mathcal{A}$ and $\overline{\mathcal{A}}$ stand respectively for the interior and the closure of $\mathcal{A}$. Given a vector x in $\mathbb{R}^n$, $x^\top$ stands for the transpose of x. The Lie derivative of a function V with respect to the vector f, i.e., $\nabla V \cdot f$ will be denoted by $L_f V$. The inequality operators $\prec, \preceq, \succ$ and $\succcurlyeq$ between vectors must be understood component by component, e.g. $x \prec y$ if and only if $x_i < y_i$ for all i where x_i and y_i are the ith components of x and y respectively. The i-th vector of the canonical basis is denoted by e_i. A function $\alpha : [0,\infty) \to \mathbb{R}$ is of class $\mathcal{K}$ if it is zero at zero, continuous and strictly increasing. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and is unbounded. A function $\rho : [0,\infty) \to \mathbb{R}$ belongs to $\mathcal{PD}$ (positive definite) if it is continuous, $\rho(s) > 0$ for all $s > 0$ and zero at zero.

2. Basic notions on hybrid systems

This section is devoted to briefly introduce basic definitions and notions on hybrid systems [6] needed to prove the main results of this paper. By definition, hybrid systems are complex dynamical systems that exhibit both continuous and discrete dynamic behavior and viewed as a set of ordinary differential equations (ODE) governed by a finite-state automaton [6]. Mathematically, these dynamical systems can be described as follows:

$$\begin{cases} \dot{x} = f(x) & \text{if } x \in \mathcal{F}, \\ x^+ \in g(x) & \text{if } x \in \mathcal{J}, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ stands for the state of (1) with the vector field $f : \mathbb{R}^n \to \mathbb{R}^n$. The set-valued mapping $g : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is the reset function of (1). The sets $\mathcal{F}$ and $\mathcal{J}$ are two closed subsets of $\mathbb{R}^n$ respectively called flow and jump sets. Note that, in this work, the design of the two event-triggered sampling algorithms is based on the flow and jump sets. We will define these sets later.

So, the hybrid dynamics involve the notion of *compact hybrid time domain* (see [6, Definition 2.3]). A set E is a compact hybrid time domain if

$$E = \bigcup_{j=0}^{J-1} ([t_j, t_{j+1}], j),$$

for some finite sequence of times $0 = t_0 \le t_1 \cdots \le t_J$. It is a *hybrid time domain* if for all $(T, J) \in E$, $E \cap ([0,T] \times \{0,1,\ldots J\})$ is a compact hybrid time domain. A solution x to (1) consists of a hybrid time domain dom x and a function $x : \text{dom}\, x \to \mathbb{R}^n$ such that $x(t,j)$ is absolutely continuous in t for a fixed j and $(t,j) \in \text{dom}\, x$ satisfying

(S1) for all $j \in \mathbb{N}$ and almost all t such that $(t,j) \in \text{dom}\, x$,

$$x(t,j) \in \mathcal{F}, \quad \dot{x}(t,j) = f(x(t,j)),$$

(S2) for all $(t,j) \in \text{dom}\, x$ such that $(t, j+1) \in \text{dom}\, x$,

$$x(t,j) \in \mathcal{J}, \quad x(t,j+1) \in g(x(t,j)).$$

When the state $x(t,j)$ belongs to the intersection of the flow set and of the jump set, then the solution can either flow or jump. Let us emphasize that the state of (1) should be either in $\mathcal{F}$ or in $\mathcal{J}$, and there is no solution issuing from $\mathbb{R}^n \backslash (\mathcal{F} \cup \mathcal{J})$.

A solution x to (1) is said to be *complete* if its domain is unbounded (either in the t-direction or in the j-direction), *Zeno* if it is complete but the projection of dom x onto $\mathbb{R}_{\ge 0}$ is bounded, and *maximal* if there does not exist another solution $\tilde{x}$ to (1) such that x is a truncation of $\tilde{x}$ to some proper subset of its domain. Hereafter, only maximal solutions will be considered.

In the literature (see, e.g., [6, Definition 3.6]), one associates to the hybrid system (1) the following stability definition.

Definition 1. Let $\mathcal{A}$ be a closed subset of $\mathbb{R}^n$ and $\mathcal{H}$ be the hybrid system defined in (1). The set $\mathcal{A}$ is said to be

- stable for $\mathcal{H}$: if for each $\epsilon > 0$ there exists $\delta > 0$ such that each solution x to $\mathcal{H}$ with $|x(0,0)|_{\mathcal{A}} \le \delta$ satisfies $|x(t,j)|_{\mathcal{A}} \le \epsilon$ for all $(t,j) \in \text{dom}\, x$;
- pre-attractive for $\mathcal{H}$: if all complete solutions satisfy $\lim_{t+j\to\infty} |x(t,j)|_{\mathcal{A}} = 0$;
- globally pre-asymptotically stable for $\mathcal{H}$: if it is both stable and pre-attractive for $\mathcal{H}$;
- globally asymptotically stable for $\mathcal{H}$: if it is globally pre-asymptotically stable for $\mathcal{H}$ and if each solution to $\mathcal{H}$ is complete.

3. Problem statement

Consider a nonlinear system

$$\dot{x}_p = f_p(x_p, u), \tag{2}$$

where f_p: $\mathbb{R}^{n_p} \times \mathbb{R}^m \to \mathbb{R}^{n_p}$ is continuously differentiable, x_p stands for the state of the plant and u stands for the control.

Assume that there exists a continuous state feedback control law $u = k(x_p)$ for which system (2) in closed loop with k is globally asymptotically stable. Then, the aim of this work is to design event-based sampling algorithms for the stabilizing state feedback control $u = k(x_p)$ by combining reachability analysis with stability analysis of hybrid systems. These sampling algorithms depend on the state of two auxiliary autonomous systems:

$$\dot{x}_a = f_a(x_a), \tag{3a}$$

$$\dot{x}_b = f_b(x_b), \tag{3b}$$

as illustrated in Fig. 1. In (3), $f_a : \mathbb{R}^{n_a} \to \mathbb{R}^{n_a}$ and $f_b : \mathbb{R}^{n_b} \to \mathbb{R}^{n_b}$ are two continuously differentiable functions.

So, the closed-loop system presented in Fig. 1 is more formally written as a hybrid system $\mathcal{H}$:

$$\mathcal{H} : \begin{cases} \begin{cases} \dot{x}_p = f_p(x_p, s) \\ \dot{x}_a = f_a(x_a) \\ \dot{x}_b = f_b(x_b) \\ \dot{s} = 0 \end{cases}, & x \in \mathcal{F}, \\ \begin{cases} x_p^+ \in \{x_p\} \\ x_a^+ \in k_a(x_a, x_p) \\ x_b^+ \in k_b(x_b, x_p) \\ s^+ \in \{k(x_p)\} \end{cases}, & x \in \mathcal{J}, \end{cases} \tag{4}$$

where $x = (x_p^\top, x_a^\top, x_b^\top, s^\top)^\top$ in $\mathbb{R}^n$ stands for the state of this

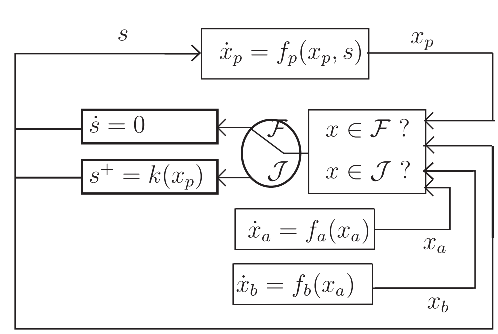

Fig. 1. Event-based sampling algorithm for the state feedback controller $k(x_p)$.