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Null controllability of neutral system with infinite delays

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ABSTRACT

Sufficient conditions are developed for the null controllability of neutral control systems with infinite delays when the values of the control lie in an m-dimensional unit cube. Conditions are placed on the perturbation function which guarantee that if the uncontrolled system is uniformly asymptotically stable and the control system satisfies a full rank condition, so that $K(\lambda)\xi(\exp(-\lambda h)) \neq 0$, for every complex λ, where $K(\lambda)$ is an $n \times n$ polynomial matrix in λ constructed from the coefficient matrices of the control system and $\xi(\exp(-\lambda h))$ is the transpose of $[1, \exp(-\lambda h), \cdots, \exp(-(n-1)\lambda h)]$, then the control system is null controllable with constraint.

1. Introduction

Controllability is one of the most important structural properties of dynamical systems used to design model based controllers and estimators. Neutral functional differential systems provide a useful modelling framework for a range of applications in science and engineering, see [3,12] and references therein. The controllability of neutral functional differential systems in particular, null controllability has been studied by many researchers including [14] and [21]. Onwuatu [14] established sufficient conditions for null controllability in function space for a class of linear and nonlinear neutral systems by showing that if the uncontrolled systems are uniformly asymptotic stable and the linear control base systems are controllable, then the null controllability of the systems is guaranteed in function space with constraints, provided that nonlinearities satisfy the necessary conditions imposed on them. In [21] it was shown that, the null controllability of a class of nonlinear neutral system is implied by the null controllability of its linear approximations under very general conditions for which the analogous result is not in general true for retarded systems. The condition under which null controllability of the system implies local controllability was also demonstrated. These studies have been extended to linear and nonlinear neutral functional differential systems with infinite delays [15,1,19,20,5,7]. The methods employed to address this problem include fixed point theorems [5], non-singularity of the controllability grammiam [19], and rank criterion for properness [7]. Controllability results can be established with the controls assumed to be either restrained or unrestrained but is required only to be square integrable on finite intervals [2]. In the latter case, a non-singularity assumption for the controllability matrix of the system is a necessary and sufficient condition for null controllability. In the former case, such conditions are however no longer sufficient for null controllability and an additional condition of stability for the uncontrolled system is required [2]. In Dauer et al. [5], null controllability result is obtained by using the Schauder fixed point theorem based on the uniform asymptotic stability of the uncontrolled system and properness assumption on the linear control system, the latter being equivalent to the non-singularity of controllability matrix. However, evaluating controllability analytically for linear time varying systems unlike time-invariant system is challenging even for very simple systems since it involves the evaluation of the controllability matrices. The controllability matrix may be calculated by computational methods provided that all the exact time-varying elements in the linear time varying systems are known.

In this paper, the controls are assumed to be restrained and the null controllability result is obtained using the Schauder's fixed point theorem. It extends the results from [21,10,16] to neutral functional differential system having infinite delays by using the Schauder fixed point theorem. Growth and continuity conditions are paced on the perturbation function which guarantees that if the linear control base system has full rank with the condition that $K(\lambda)\xi(\exp(-\lambda h)) \neq 0$ for every complex λ, where $K(\lambda)$ is an $n \times n$ polynomial matrix in λ constructed from the coefficient matrices

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of the control system and $\xi(\exp(-\lambda h))$ is the transpose of $[1, \exp(-\lambda h), \cdots, \exp(-(n-1)\lambda h)]$, and the functional difference operator for the system uniformly stable, with the linear uncontrolled system uniformly asymptotically stable, then the perturbed neutral system with infinite delay is null controllable with constraint.

The remainder of the paper is organised as follows. Section 2 contains the mathematical notations, preliminaries and problem definition. Section 3 presents the stability theorems for the system. Section 4 develops and proves the controllability theorems for the system; the main result of the paper is also developed and proved in this section. Finally, Section 5 contains numerical examples of the theoretical results prior to the discussions and conclusion.

2. Basic notations, preliminaries and definitions

Suppose $h>0$ is a given number, $E=(-\infty,\infty)$, E^n is a real n – dimensional Euclidean space with norm $|\bullet|$. Let J be any interval in E, the convention $W_2^{(0)}(J, E^n)$ represents the Lebesgue space of square-integrable functions from J to E^n, and $W_2^{(1)}([-h, 0], E^n)$ is the Sobolev space of all absolutely continuous functions $x:[-h, 0] \to E^n$ whose derivatives are square integrable. $C=C([-h, 0], E^n)$ is the space of continuous function mapping the interval $[-h, 0]$ into E^n with the norm $\|\bullet\|$, where $\|\phi\| = \sup_{-h<s\le 0}|\phi(s)|$. Define the symbol $x_t \in C$ by $x_t(s)=x(t+s)$, $-h\le s\le 0$.

This paper will consider neutral functional differential system with infinite delays of the form

$$\left.\begin{aligned} \tfrac{d}{dt}D(t)x_t &= L(t,x_t,u)+\int_{-\infty}^{0} A(\theta)x(t+\theta)d\theta \\ x_\sigma &= \phi, t\ge\sigma \end{aligned}\right\}, \tag{1}$$

and its perturbation

$$\frac{d}{dt}D(t)x_t = L(t,x_t,u)+\int_{-\infty}^{0} A(\theta)x(t+\theta)d\theta + f(t,x_t,u(t)) \tag{2}$$

through its linear base control system

$$\frac{d}{dt}D(t)x_t = L(t,x_t,u), \tag{3}$$

and its free system

$$\frac{d}{dt}D(t)x_t = L(t,x_t,0)+\int_{-\infty}^{0} A(\theta)x(t+\theta)d\theta, \tag{4}$$

where the functional difference operator $D: E\times C\to E^n$ for the system is defined by

$D(t)x_t = x(t)-A_0(t)x(t-h)$, and $L(t,x_t,u)=A_1(t)x(t)+A_2(t)x(t-h) + B(t)u(t)$, with the following assumptions:

(i) $A_0(t)$, $A_1(t)$ and $A_2(t)$ are continuous $n\times n$ matrices
(ii) $B(t)$, is a continuous $n\times m$ matrix
(iii) $A(\theta)$ is an $n\times n$ matrix whose elements are square integrable on $(-\infty, 0]$
(iv) $f:[\sigma,\infty)\times W_2^{(1)}\times E^n\to E^n$ is a nonlinear continuous matrix function.

It is assumed that f satisfy enough smoothness conditions to ensure that a solution of (2) exists through each (σ,ϕ), is unique, and depends continuously upon (σ,ϕ) and can be extended to the right as long as the trajectory remains in a bounded set $[\sigma,\infty)\times C$. These conditions are given in [4].

Let $x(\sigma,\phi)$ be a solution of (3) with $u=0$ and the set $T(t,\sigma)\phi = x_t(\sigma,\phi)$, $\phi\in W_2^{(1)}$. Then $T(t,\sigma)$ is a continuous linear operator from $W_2^{(1)}\to W_2^{(1)}$. There is an $n\times n$ matrix function $X(t,s)$ which is defined on $0\le t\le s$, $t\in J=[\sigma,\infty)$, continuous in s from the right of bounded variation in s; $X(t,s)=0, t<s\le t_1$, such that $X(t,s)$ satisfies

$$\frac{\partial D(t)X(t,s)}{\partial s} = L(t, X_t(\bullet,s),0),\ t\ge s.$$

Now, define the $n\times n$ matrix function X_0 as

$$X_0(s) = \begin{cases} 0, & -h\le s<0 \\ I, & s=0 \end{cases}$$

where I is the identity matrix. Write $T(t,s)X_0(s) = X(t+\sigma,s) = X_t(\bullet,s)(s)$, so that $T(t,s)I = X(t,s)$. A solution x of (3) through (σ,ϕ) satisfies the equation $x_t(\sigma,\phi,u) = T(t,\sigma)\phi + \int_\sigma^t T(t,\sigma)X_0B(s)u(s)ds$,

or

$$x_t(\sigma,\phi,u) = x_t(\sigma,\phi,0)+\int_\sigma^t X(t,s)B(s)u(s)ds. \tag{5}$$

In a similar manner, any solution of system (2) will be given by

$$x_t(\sigma,\phi,u,f) = x_t(\sigma,\phi,0)+\int_\sigma^t X(t,s)B(s)u(s)ds + \int_\sigma^t X(t,s)\int_{-\infty}^{0} A(\theta)x(t+\theta)d\theta\, ds$$
$$+\int_\sigma^t X(t,s)f(s,x_s,u(s))ds, \tag{6}$$

Define the matrix functions Z by

$$Z(t,s) = X(t,s)B(s), \tag{7}$$

for $t\ge s\ge\sigma$, it follows then from (6) that

$$x_t(\sigma,\phi,u,f) = x_t(\sigma,\phi,0)+\int_\sigma^t Z(t,s)u(s)ds + \int_\sigma^t X(t,s)\int_{-\infty}^{0} A(\theta)x(t+\theta)d\theta\, ds$$
$$+\int_\sigma^t X(t,s)f(s,x_s,u(s))ds. \tag{8}$$

The controls of interest, denoted by U, will be functions $u:[\sigma,\infty)\to C^m$ which are square integrable on finite intervals with values in C^m, where

$C^m = \{u: u\in E^m, |u_j|\le 1, j=1,2,\cdots,m\}$.

Definition 1. The system (3) is proper on $[\sigma,t_1]$ if $\eta^T Z(t_1,s)=0$ almost everywhere $s\in[\sigma,t_1]$ implies $\eta=0$ for $\eta\in E^n$, where η^T is the transpose of η. If (3) is proper on each interval $[\sigma,t_1]$, then the system is said to be proper in E^n

Definition 2. System (3) is said to be controllable on $[\sigma,t_1]$, if for each function $\phi\in W_2^{(1)}([-h,0],E^n)$, there is a control $u\in L_2([\sigma,t_1],E)$ such that the $x_{t_1}(\bullet,\sigma,0,u)=\phi$.

Definition 3. System (3) is said to be completely controllable on $[\sigma,t_1]$, if for each function $\phi\in W_2^{(1)}$, $x_1\in E^n$ there is an admissible control $u\in L_2([\sigma,t_1],E)$ such that the solution $x(t,\sigma,\phi,u)$ of (3) satisfies $x_\sigma(\bullet,\sigma,\phi,u)=\phi$, $x_{t_1}(\bullet,\sigma,\phi,u)=x_1$. It is completely controllable on $[\sigma,t_1]$ with constraints, if the above holds with $u\in U$.

Definition 4. The system (2) is null-controllable on $[\sigma,t_1]$ if for each $\phi\in W_2^{(1)}([-h,0],E^n)$, there exists a $u\in L_2([\sigma,t_1],E^m)$ such that the solution of (2) satisfies $x_\sigma(\bullet,\sigma,\phi,u,f)=\phi$,