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# $H_\infty$ dynamical observers design for linear descriptor systems. Application to state and unknown input estimation

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The present paper considers the $H_\infty$ observers design problem for descriptor systems. The goal is to design a dynamical observer such that the resulting dynamic estimation error is stable while the transfer function from the disturbance input to this error satisfies a prescribed $H_\infty$ norm level. Conditions for the solvability of this problem are obtained in terms of a set of linear matrix inequalities (LMIs). An extension to a simultaneous estimation of state and unknown input for descriptor systems is presented. A numerical example is provided to illustrate our approach.



## 1. Introduction

This paper concerns the $H_\infty$ dynamical observers design for descriptor systems. Descriptor systems, also known as singular or differential algebraic systems, can be considered as a generalization of dynamical systems. Descriptor system representation is a powerful modeling tool since it can describe processes governed by differential and algebraic equations. It represents the physical phenomena that the model by ordinary differential equations can not describe. These systems were introduced by [19] from a control theory point of view and since, a large amount of works have been published dealing with analysis and control of them (see [5,21,15,6,7,23,24]).

Very often physical systems are subject to uncertainties, disturbances or noises that affect the nominal system, and consequently their state estimation. The uncertainties can result from system identification, model reduction, time delays and non-linearities [29].

One of the most popular ways to deal with this problem is to use the Kalman filter which needs statistical informations about the noises, however, in some applications the noise sources may not be exactly known. Also Kalman filtering cannot guarantee satisfactory performances when there are parameter uncertainties in a system model (see [4,10,12,16,22]).

In view of these, an alternative estimation method based on $H_\infty$ observer has been proposed recently. The advantage of the $H_\infty$ observer is that it does not require any knowledge on the statical properties of disturbances, the latter must only be of bounded energy [2,31]. Recently a numerous works have been appeared that deal with the $H_\infty$ observers design for descriptor systems [6,8,17,27,28], all these results use the Proportional Observer (PO).

Another kind of observer is the Proportional Integral Observers (PIO), it has been introduced by duality to Proportional Integral (PI) controller which are generally used to achieve steady-state accuracy and are also useful for the control of unknown systems. The PIO constitutes an extension of the Luenberger observers (called also PO). A new structure of observers called dynamic observers was developed by [13,20], it presents an alternative state estimation approach which can be considered as more general than the PO and the PIO, the latter are only particular cases of this structure. The idea of including additional dynamics in the observer was presented in [13].

On the other hand the design of unknown input observer is a crucial problem since in many practical cases all input signals cannot be known. For the case of standard systems, one of the approaches most used to deal with unknown input estimation is the adaptive observer design (see [18] and the special issue on fault diagnosis and fault tolerant control [30]). However, for the descriptor systems case, only few works deal with this problem [1]. Our approach is different

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from adaptive observer approach, since it does not use an adaptive law for the unknown input estimation. It estimates simultaneously the state and the unknown input in the descriptor systems case.

The main contribution of this paper is that the designed observer is presented in a more general form than the existing dynamical observers, the PO and PIO are only particular cases of the structure of our observer. The structure of the dynamical observer contains an additional variable which represents the integral of the state of the observer and the output. It is a generalization of the integral of the output error (difference between the estimated output and the output). Its role is to increase steady state accuracy and improve robustness in estimation performances against disturbances and modeling errors. It also provides additional degrees of freedom in the observer design which can be used to increase the stability margin. Previous work on this approach can be seen in [23] with preliminaries results. Finally the effectiveness of this approach is shown through a numerical example.

## 2. Preliminaries

In this section we shall present the notation and some basic results which are used in the sequel of the paper. The set of real matrices $n \times m$ is denoted by $\mathbb{R}^{n \times m}$. $A^T$ denotes the transpose of the matrix $A$. $I_n$ denotes an $n \times n$ identity matrix, $I$ denotes an identity matrix with appropriate dimension, 0 denotes a zero matrix of appropriate dimension. The symbol $(\ast)$ denotes the transpose elements in the symmetric positions, $\text{ones}_{n,m}$ denotes a matrix of dimension $n \times m$ with all elements equal to one. $\|f(t)\|_2 = \sqrt{\int_0^\infty \|f(t)\|^2\, dt}$, $f(t) \in L_2[0,\infty)$, where $L_2[0,\infty)$ stands for the space of square integrable functions on $[0,\infty)$ and $\|f(t)\|$ means the Euclidean vector of function $f(t)$ at time $t$. The symbol $A^\perp$ denotes a maximal row rank matrix such that $A^\perp A = 0$, by convention $A^\perp = 0$ when $A$ is of full row rank. The symbol $A^+$ denotes any generalized inverse of the matrix $A$, satisfying $AA^+A = A$.

**Remark 1** (*Bernstein [3]*). Let $A \in \mathbb{R}^{n \times m}$. If $\text{rank}(A) = m$, then $A^+$ is a left inverse of $A$, it satisfies $A^+A = I$. If $\text{rank}(A) = n$, then $A^+$ is a right inverse of $A$, it satisfies $AA^+ = I$. Both left and right inverses satisfy $AA^+A = A$.

The following lemma is used in the sequel of the paper.

**Lemma 1** (*Skelton et al. [25]*). *Let matrices $\mathcal{B}, \mathcal{C}, \mathcal{D} = \mathcal{D}^T$ be given, then the following statements are equivalent:*

(S1) *There exists a matrix $\mathcal{X}$ satisfying*
$$\mathcal{B}\mathcal{X}\mathcal{C} + (\mathcal{B}\mathcal{X}\mathcal{C})^T + \mathcal{D} < 0$$

(S2) *The following two conditions hold*
$$\mathcal{B}^\perp \mathcal{D} \mathcal{B}^{\perp T} < 0 \quad \text{or } \mathcal{B}\mathcal{B}^T > 0$$
$$\mathcal{C}^{T\perp} \mathcal{D} \mathcal{C}^{T\perp T} < 0 \quad \text{or } \mathcal{C}^T\mathcal{C} > 0$$

*Suppose that the statement (S2) holds. Let $r_b$ and $r_c$ be the ranks of $\mathcal{B}$ and $\mathcal{C}$, respectively, and $(\mathcal{B}_l, \mathcal{B}_r)$ and $(\mathcal{C}_l, \mathcal{C}_r)$ be any full rank factors of $\mathcal{B}$ and $\mathcal{C}$, i.e. $\mathcal{B} = \mathcal{B}_l\mathcal{B}_r$, $\mathcal{C} = \mathcal{C}_l\mathcal{C}_r$. Then the matrix $\mathcal{X}$ in statement (S1) is given by*

$$\mathcal{X} = \mathcal{B}_r^+ \Delta \mathcal{C}_l^+ + \Theta - \mathcal{B}_r^+ \mathcal{B}_r \Theta \mathcal{C}_l \mathcal{C}_l^+$$

*where $\Theta$ is an arbitrary matrix and*

$$\Delta = -\mathcal{R}^{-1}\mathcal{B}_l^T \Lambda \mathcal{C}_r^T (\mathcal{C}_r \Lambda \mathcal{C}_r^T)^{-1} + \Gamma^{1/2} \Psi (\mathcal{C}_r \Lambda \mathcal{C}_r^T)^{-1/2}$$

$$\Gamma = \mathcal{R}^{-1} - \mathcal{R}^{-1}\mathcal{B}_l^T \left[\Lambda - \Lambda \mathcal{C}_r^T (\mathcal{C}_r \Lambda \mathcal{C}_r^T)^{-1} \mathcal{C}_r \Lambda\right] \mathcal{B}_l \mathcal{R}^{-1}$$

*where $\Psi$ is an arbitrary matrix such that $\|\Psi\| < 1$ and $\mathcal{R}$ is an arbitrary positive definite matrix such that*

$$\Lambda = (\mathcal{B}_r \mathcal{R}^{-1} \mathcal{B}_l^T - \mathcal{D})^{-1} > 0.$$

## 3. Problem formulation

Consider the descriptor system described by

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + D_1 w(t) \\ y(t) &= Cx(t) + D_2 w(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the semi-state vector, $u(t) \in \mathbb{R}^m$ is the known input, $w(t) \in \mathbb{R}^{n_w}$ is the disturbance vector of bounded energy and $y(t) \in \mathbb{R}^p$ represents the measured output vector. Matrix $E \in \mathbb{R}^{n_1 \times n}$ and when $n_1 = n$ matrix $E$ is singular. Matrices $A \in \mathbb{R}^{n_1 \times n}$, $B \in \mathbb{R}^{n_1 \times m}$, $D_1 \in \mathbb{R}^{n_1 \times n_w}$, $C \in \mathbb{R}^{p \times n}$ and $D_2 \in \mathbb{R}^{p \times n_w}$. Let $\text{rank}(E) = r < n$ and let $E^\perp \in \mathbb{R}^{r_1 \times n_1}$ be a full row rank matrix such that $E^\perp E = 0$, in this case we have $r_1 = n_1 - r$.

In the sequel we assume that

**Assumption 1.**

$$\text{rank}\begin{bmatrix} E \\ E^\perp A \\ C \end{bmatrix} = n.$$

**Remark 2.** Assumption 1 is equivalent to the impulse observability [6], i.e.

$$\text{rank}\begin{bmatrix} E & A \\ 0 & C \\ 0 & E \end{bmatrix} = n + \text{rank}(E).$$

This condition is more general than the one ($\text{rank}\left[\begin{smallmatrix} E \\ C \end{smallmatrix}\right] = n$) generally considered, see for example [9,26,14]. Now let us consider the following dynamical observer for system (1)

$$\dot{\zeta}(t) = N\zeta(t) + Hv(t) + F\begin{bmatrix} -E^\perp Bu(t) \\ y(t) \end{bmatrix} + Ju(t) \tag{2}$$

$$\dot{v}(t) = S\zeta(t) + Lv(t) + M\begin{bmatrix} -E^\perp Bu(t) \\ y(t) \end{bmatrix} \tag{3}$$

$$\hat{x}(t) = P\zeta(t) + Q\begin{bmatrix} -E^\perp Bu(t) \\ y(t) \end{bmatrix} \tag{4}$$

where $\zeta \in \mathbb{R}^q$ represents the state vector of the observer, $v(t) \in \mathbb{R}^\nu$ is an auxiliary vector and $\hat{x}(t) \in \mathbb{R}^n$ is the estimate of $x(t)$. Matrices $N, F, J, H, L, M, S, P$ and $Q$ are unknown and of appropriate dimensions which must be determined such that $\hat{x}(t)$ asymptotically converges to $x(t)$ for $w(t) = 0$, and for $w(t) \neq 0$ we must satisfy $\min \|T_{we}\|_\infty = \min \sup_{w \in L_2 - \{0\}} \frac{\|e\|_2}{\|w\|_2} < \gamma$, where $T_{we}$ is the transfer function from the disturbance to the estimation error and $\gamma$ is a given positive scalar.

Now we can give the following lemma.

**Lemma 2.** *There exists an observer of the form (2)–(4) for system (1) if and only if the following two statements hold*

(1) *There exists a matrix $T$ of appropriate dimension such that the following conditions are satisfied*

(a) $NTE + F\begin{bmatrix} E^\perp A \\ C \end{bmatrix} - TA = 0$