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Plug-and-play state estimation and application to distributed output-feedback model predictive control ☆

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ABSTRACT

In this paper we propose a novel distributed state estimator for large-scale linear systems composed by subsystems interacting through state variables. The distributed state estimator has the following features: (i) local state estimators, each dedicated to the reconstruction of the states of a subsystem, are connected through a communication network with the parent–child topology induced by subsystems coupling; (ii) the design of a local state estimator requires information on the associated subsystem and its parents only. As a consequence, both the offline design and the online implementation are distributed and scalable. In particular, the addition and removal of subsystems can be handled in a plug-and-play fashion. The distributed state estimator is also combined with a plug-and-play distributed model predictive control scheme to provide a novel output-feedback plug-and-play distributed controller capable of guaranteeing nominal convergence and constraint satisfaction. Applications to a mechanical system and power networks demonstrate the effectiveness of the approach.

1. Introduction

In the last years, we have witnesses a renewed interest in the development of distributed and decentralized control and state estimation methods [25,3]. This has been motivated by the ever increasing need of coordination and integration of a possibly large number of different devices. When a plant is composed of several interconnected subsystems, centralized control and state estimation architectures are inadequate. In fact, online operations, such as the transmission of measurements to a central processing unit or the simultaneous estimation of all states, can be prohibitive.

Focusing on state estimation, a large body of research has been recently devoted to the development Distributed State Estimators (DSEs) where subsystems are equipped with Local State Estimators (LSEs) connected through a communication network. While in some approaches each LSE has to reconstruct the state of the overall system (see, e.g., [2,26] and references therein for theoretical contributions and applications to power networks), in other methods LSEs are dedicated to the reconstruction of local states only [24,38,12,34,35,4,7,17,21]. In this paper we consider the latter case. In terms of communication requirements, some methods are more parsimonious than others, as they do not require *all-to-all information exchange* among LSEs. For instance, some DSEs require only a network matching the *parent–child topology* due to coupling among subsystems [12,34,35,4,7,17,21]. Furthermore, there are methods that also guarantee the fulfillment of constraints on local states [4] or estimation errors [7,17,21].

To the best of our knowledge, the DSE proposed in [21] is the first one relying on a fully distributed design procedure. In [21], boundedness and convergence of the global estimation error can be guaranteed through numerical tests that are associated with individual LSEs and that can be performed in parallel using hardware collocated with subsystems. Moreover, each test involves information about a subsystem and its parents only, hence requiring a communication network with the same parent–child topology used for real-time operations. Reconfiguration of the estimation scheme can be performed through Plug-and-Play (PnP) design, meaning that (i) when a subsystem is added to a plant, the corresponding LSE can be designed using pieces of information from parent subsystems only; (ii) in order to preserve the key properties of the whole DSE, the plugging in and out of a subsystem triggers at most the update of LSEs associated to child subsystems and (iii) the design/update of an LSE is automatized, e.g. as in our approach it is recast into an optimization problem that can be solved using local hardware. We argue that PnP design (firstly introduced in [37] for control) can be useful in the context of systems of systems [32] and cyber-physical systems [1] where, typically, the number of subsystems changes over time.

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In this paper we extend the results presented in [21] to account for bounded measurement noise. Furthermore, we use the DSE in combination with the PnP distributed Model Predictive Control (MPC) schemes described in [30,22] to provide a novel output-feedback PnP distributed control algorithm. More specifically, we show how the tube MPC scheme in [14] can be used in PnP design to guarantee closed-loop nominal convergence and constraint satisfaction at all times.

Recently, output feedback Distributed MPC (DMPC) schemes have been proposed based on a cooperative approach (see, e.g., [36,10]) or in a non-cooperative setting (see, e.g., [7,31]). However, in contrast with PnP design, all these methods involve a centralized offline design phase.

This paper is structured as follows. In Section 2 we introduce the state estimation problem and the main assumptions on the system, while in Section 3 we describe the PnP DSE. In Section 4 we describe the PnP output-feedback DMPC strategy embedding the proposed estimation scheme. In Section 5 two case studies are discussed: the application of DSE to an array of 16 masses connected by springs and dampers, and output-feedback frequency control in a power network. In Section 6 some conclusions are drawn.

Notation: We use $a:b$ for the set of integers $\{a, a+1, \dots, b\}$. The column vector with s components $v_1, \dots, v_s$ is $\mathbf{v} = (v_1, \dots, v_s)$. The symbol $\oplus$ denotes the Minkowski sum, i.e. $A = B \oplus C$ if and only if $A = \{a : a = b + c, b \in B, c \in C\}$. Moreover, $\oplus_{i=1}^{s} G_i = G_1 \oplus \cdots \oplus G_s$. The Pontryagin difference is denoted by $\ominus$, i.e. $A = B \ominus C$ iff $A = \{a : a + c \in B, \forall c \in C\}$. The symbol $\mathbf{1}_\alpha$ (resp. $\mathbf{0}_\alpha$) denotes a column vector with $\alpha \in \mathbb{N}$ elements all equal to 1 (resp. 0). The identity matrix of size n is $\mathbb{I}_n$. Given a matrix $A \in \mathbb{R}^{n \times n}$, with entries a_{ij} its entry-wise 1-norm is $||A||_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}|$ and its Frobenius norm is $||A||_F = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2}$. The standard Euclidean norm is denoted with $\| \cdot \|$. The pseudo-inverse of a matrix $A \in \mathbb{R}^{m \times n}$ is denoted with $A^\flat$. A matrix $A \in \mathbb{R}^{n \times n}$ is Schur stable if all its eigenvalues λ verify $|\lambda| < 1$. Moreover, for $\delta > 0$, $\mathcal{B}_\delta(v) = \{x \in \mathbb{R}^n : \|x - v\| < \delta\}$.

The set $\mathbb{X} \subseteq \mathbb{R}^n$ is Robust Positively Invariant (RPI) for $x(t+1) = f(x(t), w(t))$, $w(t) \in \mathbb{W} \subseteq \mathbb{R}^m$ if $x(t) \in \mathbb{X} \Rightarrow f(x(t), w(t)) \in \mathbb{X}$, $\forall w(t) \in \mathbb{W}$.

2. Model of interconnected systems

We consider a discrete-time linear time-invariant system described by

$$\mathbf{x}^+ = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d} \tag{1a}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \boldsymbol{\varrho} \tag{1b}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$, $\mathbf{y} \in \mathbb{R}^p$, $\mathbf{d} \in \mathbb{R}^r$ and $\boldsymbol{\varrho} \in \mathbb{R}^p$ are the state, the input, the output, the model disturbance and the output disturbance, respectively, at time t and $\mathbf{x}^+$ stands for $\mathbf{x}$ at time $t+1$. Let $\mathcal{M} = 1:M$ be the set of subsystem indexes. We assume that the state is composed of M state vectors $x_{[i]} \in \mathbb{R}^{n_i}$, $i \in \mathcal{M}$ such that $\mathbf{x} = (x_{[1]}, \dots, x_{[M]})$, and $n = \sum_{i \in \mathcal{M}} n_i$. Similarly, the input, the output, the model disturbance and the output disturbance are composed of vectors $u_{[i]} \in \mathbb{R}^{m_i}$, $y_{[i]} \in \mathbb{R}^{p_i}$, $d_{[i]} \in \mathbb{R}^{r_i}$, $\varrho_{[i]} \in \mathbb{R}^{p_i}$, $i \in \mathcal{M}$ such that $\mathbf{u} = (u_{[1]}, \dots, u_{[M]})$, $m = \sum_{i \in \mathcal{M}} m_i$, $\mathbf{y} = (y_{[1]}, \dots, y_{[M]})$, $p = \sum_{i \in \mathcal{M}} p_i$, $\mathbf{d} = (d_{[1]}, \dots, d_{[M]})$, $r = \sum_{i \in \mathcal{M}} r_i$ and $\boldsymbol{\varrho} = (\varrho_{[1]}, \dots, \varrho_{[M]})$. We assume that system (1) is partitioned into M interconnected subsystems $\Sigma_{[i]}$, $i \in \mathcal{M}$, each described by the following dynamical equations:

$$\Sigma_{[i]}: \quad x_{[i]}^+ = A_{ii} x_{[i]} + B_i u_{[i]} + \sum_{j \in \mathcal{N}_i} A_{ij} x_{[j]} + D_i d_{[i]} \tag{2a}$$

$$y_{[i]} = C_i x_{[i]} + \varrho_{[i]} \tag{2b}$$

where $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $i, j \in \mathcal{M}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, $D_i \in \mathbb{R}^{n_i \times r_i}$, $C_i \in \mathbb{R}^{p_i \times n_i}$. Since, in general, $A_{ij} \neq \mathbf{0}_{n_i \times n_j}$ for $j \neq i$, the subsystems are coupled through state variables (i.e., they are dynamically coupled). We say that $\Sigma_{[j]}$ is a *parent* of $\Sigma_{[i]}$ if $A_{ij} \neq \mathbf{0}_{n_i \times n_j}$, and we define with $\mathcal{N}_i = \{j \in \mathcal{M} : A_{ij} \neq \mathbf{0}_{n_i \times n_j}, i \neq j\}$ the set of parents of $\Sigma_{[i]}$. From (2b), it is assumed that $y_{[i]}$ depends on the local state $x_{[i]}$ and the local measurement noise $\varrho_{[i]}$ only, while, from (2a), the time evolution of $x_{[i]}$ directly depends only upon the local input $u_{[i]}$ and the local disturbance $d_{[i]}$. Generalizations of model (2) are discussed in Remark 3. We also assume that subsystems $\Sigma_{[i]}$, $i \in \mathcal{M}$ are subject to the constraints

$$d_{[i]} \in \mathbb{D}_i, \quad \varrho_{[i]} \in \mathbb{O}_i. \tag{3}$$

For the overall system (1), this is equivalent to

$$\mathbf{d} \in \mathbb{D}, \quad \boldsymbol{\varrho} \in \mathbb{O}. \tag{4}$$

where $\mathbb{D} = \prod_{i \in \mathcal{M}} \mathbb{D}_i$ and $\mathbb{O} = \prod_{i \in \mathcal{M}} \mathbb{O}_i$. We introduce the following assumptions on subsystem dynamics and disturbances.

Assumption 1. (I) The pair (A_{ii}, C_i) is detectable, $\forall i \in \mathcal{M}$.

(II) Constraints $\mathbb{D}_i \subset \mathbb{R}^{r_i}$ and $\mathbb{O}_i \subset \mathbb{R}^{p_i}$ are zonotopes centered at the origin. Without loss of generality [33], $\mathbb{D}_i$ can be written as

$$\begin{aligned} \mathbb{D}_i &= \{d_{[i]} \in \mathbb{R}^{r_i} : \mathcal{F}_i w_{[i]} \leq \mathbf{1}_{\overline{v}_i}\} \\ &= \{d_{[i]} \in \mathbb{R}^{r_i} : d_{[i]} = \Delta_i F_i{}^d, \; ||F_i{}^d||_\infty \leq 1\} \end{aligned} \tag{5}$$

where $\mathcal{F}_i = (f_{i,1}^T, \dots, f_{i,\overline{v}_i}^T) \in \mathbb{R}^{\overline{v}_i \times r_i}$, $\text{rank}(\mathcal{F}_i) = r_i$, $\Delta_i \in \mathbb{R}^{r_i \times \overline{r}_i}$, $F_i^d \in \mathbb{R}^{\overline{r}_i}$. Furthermore, $\mathbb{O}_i$ can be written as

$$\begin{aligned} \mathbb{O}_i &= \{\varrho_{[i]} \in \mathbb{R}^{p_i} : \mathcal{G}_i \varrho_{[i]} \leq \mathbf{1}_{\overline{\nu}_i}\} \\ &= \{\varrho_{[i]} \in \mathbb{R}^{p_i} : \varrho_{[i]} = \Upsilon_i F_i{}^\varrho, \; ||F_i{}^\varrho||_\infty \leq 1\} \end{aligned} \tag{6}$$

where $\mathcal{G}_i = (g_{i,1}^T, \dots, g_{i,\overline{\nu}_i}^T) \in \mathbb{R}^{\overline{\nu}_i \times p_i}$, $\text{rank}(\mathcal{G}_i) = p_i$, $\Upsilon_i \in \mathbb{R}^{p_i \times \overline{p}_i}$ and $F_i^\varrho \in \mathbb{R}^{\overline{p}_i}$.

Next we introduce assumptions that will be instrumental for developing DMPC schemes.

Assumption 2. (I) The pair (A_{ii}, B_i) is stabilizable, $\forall i \in \mathcal{M}$.

(II) Subsystems $\Sigma_{[i]}$, $i \in \mathcal{M}$ are subject to the constraints

$$x_{[i]} \in \mathbb{X}_i, \quad u_{[i]} \in \mathbb{U}_i \tag{7}$$

where $\mathbb{X}_i$ and $\mathbb{U}_i$ are polytopes given by

$$\mathbb{X}_i = \{x_{[i]} \in \mathbb{R}^{n_i} : c_{x_{i,\tau}}^T x_{[i]} \leq 1, \forall \tau \in 1:\tau_i^x\} \tag{8a}$$

$$\mathbb{U}_i = \{u_{[i]} \in \mathbb{R}^{m_i} : c_{u_{i,\tau}}^T u_{[i]} \leq 1, \forall \tau \in 1:\tau_i^u\}, \tag{8b}$$

with $c_{x_{i,\tau}} \in \mathbb{R}^{n_i}$ and $c_{u_{i,\tau}} \in \mathbb{R}^{m_i}$.

3. DSE with PnP features

3.1. State estimation scheme

In this section we propose a DSE for system (1) similar to the one presented in [21], where measurement noise was not accounted for.

For each subsystem $\Sigma_{[i]}$, $i \in \mathcal{M}$, the corresponding LSE $\tilde{\Sigma}_{[i]}$ is defined as follows

$$\tilde{\Sigma}_{[i]}: \quad \begin{aligned} \tilde{x}_{[i]}^+ &= A_{ii}\tilde{x}_{[i]} + B_i u_{[i]} - L_{ii}(y_{[i]} - C_i \tilde{x}_{[i]}) \\ &+ \sum_{j \in \mathcal{N}_i} A_{ij}\tilde{x}_{[j]} - \sum_{j \in \mathcal{N}_i} \delta_{ij}^L L_{ij}(y_{[j]} - C_j \tilde{x}_{[j]}) \end{aligned} \tag{9}$$

where $\tilde{x}_{[i]} \in \mathbb{R}^{n_i}$ is the state estimate, $L_{ij} \in \mathbb{R}^{n_i \times p_j}$ are gain matrices and $\delta_{ij}^L \in \{0, 1\}$. In view of (9), $\tilde{\Sigma}_{[i]}$ depends on local variables (i.e., $\tilde{x}_{[i]}$, $u_{[i]}$ and $y_{[i]}$), and on outputs and state estimates of the parent subsystems (i.e., $\tilde{x}_{[j]}$ and $y_{[j]}$, $j \in \mathcal{N}_i$). Variables δ_{ij}^L, $j \in \mathcal{N}_i$ are set to one if parents' outputs are used for local estimation purposes, at the price of slightly increasing the amount of transmitted information. Defining

$$e_{[i]} = x_{[i]} - \tilde{x}_{[i]}, \tag{10}$$