European Journal of Control 25 (2015) 39–50

Contents lists available at ScienceDirect

European Journal of Control

journal homepage: www.elsevier.com/locate/ejcon

Application of algebraic geometry techniques in permanent-magnet DC motor fault detection and identification

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ARTICLE INFO

Article history:
Received 24 October 2014
Received in revised form
10 April 2015
Accepted 12 May 2015
Recommended by A. Astolfi
Available online 22 May 2015

Keywords:
Fault detection
DC motor
Algebraic geometry

ABSTRACT

This paper presents a fault detection method, which is able to identify the occurrence of a fault and, possibly, its nature, in permanent-magnet DC motors. Techniques arising from Algebraic Geometry are used to compute the motor dynamical model parameters in function of the output and its time derivatives, up to a finite order. Similar tools are used to compute auxiliary expressions of the state of the motor, independent of the estimated parameter. Simulative and experimental examples show the effectiveness of the proposed technique.

1. Introduction

In industrial applications, fault detection and identification are among the most important techniques to improve the safety and efficiency of many manufacturing processes [17,33,22,19,34,7,16,15,32].

Several different techniques have been developed to detect the occurrence of a fault and to determine the exact location of the fault [8]. Some of the most widespread methods used to reach such a goal are based on parity relations from the state-space model [5], on parity relations from the input–output model [14] and on state observers [3].

In this paper, attention is focused on fault detection for permanent-magnet DC motors. Such electromechanical systems are used in many high-risk industrial applications, due to their low price and to their low power consumption. Several different techniques have been developed to analyze the faults of DC motors [1,13,21,4]. Fault detection methods are generally divided into two classes: methods based on signal analysis and methods based on motor dynamical models. The first class is composed of methods based on analyzing directly the measured signal to obtain information about the state of the motor [20,12,31,10]. The second class makes use of the motor dynamical model and is based on estimating the parameters of such a motor model [11,18] or on motor state estimation [37].

The fault detection and identification method presented in this paper belongs to the class of the methods making use of the motor dynamical model. In this paper, it is assumed that

(1) the motor dynamical behavior can be described, both in the case of a normally working motor and in the case of a faulty motor, by the same nonlinear dynamical model, but with different parameters (in the following, normally working motor model parameters are called *nominal parameters*, whereas faulty motor model parameters are called *faulty parameters*);

(2) only a single faulty parameter is different from the correspondent nominal one, i.e., an admissible fault is characterized by the change of one single parameter (the possibility of removing this assumption is discussed in the conclusions of this paper).

The organization of the paper is as follows: first, by using Algebraic Geometry techniques [9,28,35,25,26,29], an algorithm is given, at the end, to compute *embeddings* (i.e., functions of the output and its time derivatives, which vanish identically along the trajectories of the system) for the motor dynamical model, both in the faulty and in the normal working case. Moreover, formulas, expressing the motor dynamical model faulty parameters and the unmeasured state, in function of the output and its time derivatives, up to a finite order, are given.

Then, such embeddings and functions are given as input to another procedure, able to identify when a fault has occurred, its nature, fault size and the unmeasured state of the motor system. To validate the method, the results obtained both in simulation and by using real experimental data are reported.

Finally, the extension of the proposed method to time-varying input voltages and load torques is reported, with a numerical simulation, to show how the proposed technique can be applied also in such a type of applications.

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http://dx.doi.org/10.1016/j.ejcon.2015.05.001
0947-3580/

2. Preliminaries and notation

The following dynamical model of a permanent-magnet DC motor is given in Liu et al. [23]:

$$\dot{x}_1 = L^{-1}(u - K_e x_2 - R_a x_1), \tag{1a}$$

$$\dot{x}_2 = J_1^{-1}(K_T x_1 - T_0 - T_2 - f_r x_2 - f_p x_2^2), \tag{1b}$$

$$y = x_1, \tag{1c}$$

where x_1 is the armature current, x_2 is the normalized rotational speed, L and u are the armature inductance and voltage, J_1 is the normalized inertial moment of the rotator, f_r is the friction coefficient due to the bearing lubrication condition, f_p is the friction coefficient due to aerodynamics, T_0 and T_2 are the no-load and load torque, K_e and K_T are coefficients, respectively. It is assumed that all these parameters are constant; such an assumption is removed in Section 5 for u and T_2.

Let $x = [x_1\ x_2]^\top$. Defining the vector field $f(x)$ and the function $h(x)$ as follows:

$$f(x) := \begin{bmatrix} L^{-1}(u - K_e x_2 - R_a x_1) \\ J_1^{-1}(K_T x_1 - T_0 - T_2 - f_r x_2 - f_p x_2^2) \end{bmatrix},$$

$$h(x) := x_1,$$

the *observability map of order* $k+1$, with k being a non-negative integer, is given by $y_{e,k} := O_k(x)$, where $y_{e,k} = [y_0\ \dots\ y_k]^\top$, $y_j(t) = d^j y(t)/dt^j$, $j = 0, \dots, k$, and

$$O_k(x) = \begin{bmatrix} L_f^0 h(x) \\ \vdots \\ L_f^k h(x) \end{bmatrix},$$

where $L_f^{j+1} h(x) = \partial L_f^j h(x)/\partial x f(x)$, $L_f^0 h(x) = h(x)$.

Let N be a given non-negative integer. A polynomial $p(y_{e,N})$ is said to be an *embedding* of system (1) if

$$p(O_N(x)) = 0, \quad \forall x \in \mathbb{R}^2,$$

i.e., if it vanishes identically along the output of system (1) and its time derivatives [27].

Let p be a given multi-variable polynomial. The *degree of* p, *with respect to the variable* x_i, denoted by $\deg_{x_i}(p)$, is the maximum exponent of the variable x_i in p. Let $\phi \in \mathbb{R}^n$ be a given vector. When needed, the symbol $[\phi]_i$ denotes the ith entry of vector ϕ.

3. Fault detector

In Section 3.1 some tools, borrowed from [27], are used to compute an embedding of the motor dynamical model in the nominal parameters. Then, a procedure (namely, Algorithm 1) able to compute embeddings of the motor dynamical model, which vanish even if a single parameter in the motor dynamical model changes, and two formulas, expressing the faulty parameter and the unmeasured state x_2 as functions of the output and its time derivatives, is reported. In Section 3.2, Algorithm 2, which takes as input the formulas and the embeddings obtained by using Algorithm 1, is used to design a fault detector for the DC motor.

3.1. Embeddings and expressions of the faulty parameters as functions of the output and its time derivatives

Define the nominal parameters $\overline{p}_i$, with $i = 1, \dots, 7$, as

$$\overline{p}_1 = \frac{u}{L}, \quad \overline{p}_2 = \frac{K_e}{L}, \quad \overline{p}_3 = \frac{R_a}{L},$$

$$\overline{p}_4 = \frac{K_T}{J_1}, \quad \overline{p}_5 = \frac{T_0 + T_2}{J_1}, \quad \overline{p}_6 = \frac{f_r}{J_1}, \quad \overline{p}_7 = \frac{f_p}{J_1}. \tag{2}$$

The dynamical model (1) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= \overline{p}_1 - \overline{p}_2 x_1 - \overline{p}_3 x_2 \\ \dot{x}_2 &= \overline{p}_4 x_1 - \overline{p}_5 - \overline{p}_6 x_2 - \overline{p}_7 x_2^2, \\ y &= x_1. \end{aligned} \tag{3}$$

Hence, the map $O_3(x)$ can be computed as

$$L_f^0 h(x) = x_1, \tag{4a}$$

$$L_f^1 h(x) = \overline{p}_1 - \overline{p}_2 x_1 - \overline{p}_3 x_2, \tag{4b}$$

$$L_f^2 h(x) = \rho_1(x), \tag{4c}$$

$$L_f^3 h(x) = \rho_2(x), \tag{4d}$$

where $\rho_1(x) = x_1(\overline{p}_2^2 - \overline{p}_3\overline{p}_4) + \overline{p}_3(x_2^2\overline{p}_7 + x_2(\overline{p}_2 + \overline{p}_6) + \overline{p}_5) - \overline{p}_1\overline{p}_2$ and $\rho_2(x) = (\overline{p}_2^2 - \overline{p}_3\overline{p}_4)(-x_1\overline{p}_2 - x_2\overline{p}_3 + \overline{p}_1) - \overline{p}_3(2x_2\overline{p}_7 + \overline{p}_2 + \overline{p}_6)(-x_1\overline{p}_4 + x_2(x_2\overline{p}_7 + \overline{p}_6) + \overline{p}_5)$.

To compute an embedding of system (3) in the nominal parameters, by using the procedure described in Menini and Tornambe [27], define the ideal $\mathcal{I}$ in $\mathbb{R}[x_1, x_2, y_2, y_1, y_0]$,

$$\mathcal{I} = \langle y_0 - L_f^0(x),\ y_1 - L_f^1(x),\ y_2 - L_f^2(x) \rangle \subset \mathbb{R}[x_1, x_2, y_2, y_1, y_0],$$

where $L_f^0 h$, $L_f^1 h$ and $L_f^2 h$ are given in (4) (note that the third order directional derivative $L_f^3 h(x)$ of (4d) is not used here).

The *Elimination Theorem* (see Cox et al. [6]) implies that, by computing a *Groebner basis* $\mathcal{G}$ of the ideal $\mathcal{I}$, according to *lexicographic ordering*, with $x_1 > x_2 > y_2 > y_1 > y_0$, a Groebner basis of the *elimination ideal* of $\mathcal{I}$, which eliminates x_1 and x_2, is given by $\mathcal{G} \cap \mathbb{R}[y_2, y_1, y_0] = \{q_n(y_{e,2})\}$, where

$$\begin{aligned} q_n(y_{e,2}) = {} & \overline{p}_3(y_1\overline{p}_2 + \overline{p}_3(y_0\overline{p}_4 - \overline{p}_5) + \overline{p}_6(y_0\overline{p}_2 \\ & - \overline{p}_1 + y_1) + y_2) - \overline{p}_7(y_0\overline{p}_2 - \overline{p}_1 + y_1)^2, \end{aligned}$$

is an embedding of (1) in the nominal parameters.

In the rest of this section, it is assumed that the dynamical behavior of the faulty motor can be described by (3) and that only a single parameter can change in such a model, due to a fault (i.e., the dynamical behavior of the faulty motor can be described by (3), by substituting a single p_i to the correspondent $\overline{p}_i$, with $i \in \{1, \dots, 7\}$).

Algorithm 1, which is based on the procedures described in [24], can be used to compute embeddings of (3), which are not explicitly dependent on the parameter $\overline{p}_i$. Hence, such embeddings vanish even if the fault, characterized by the substitution of $\overline{p}_i$ by p_i, occurs. Algorithm 1 also produces two polynomial functions which express the parameter p_i and the state variable x_2 as functions of the output and of its time derivatives.

Remark 1. To let the reader better appreciate Algorithm 1, a brief summary of the operations which have to be carried out to complete such an algorithm is reported. In Step 1, the ideal $\mathcal{I}_{p_i}$ is defined from the observability map $O_3(x)$. In Step 2, a Groebner basis $\mathcal{G}_{p_i}$ of the ideal $\mathcal{I}_{p_i}$ is computed according to the lexicographic ordering with $x_1 > x_2 > p_i > y_3 > y_2 > y_1 > y_0$. Hence, in Step 3, it is possible to compute a Groebner basis of the elimination ideal of $\mathcal{I}_{p_i}$, which eliminates x_1, x_2 and p_i, whence, by such a basis, in Step 4, an embedding of system (1), which vanishes along the trajectories of system (1), even when the parameter p_i changes, can be computed. In Step 5, always by the Elimination Theorem (see Cox et al. [6]), a Groebner basis of the elimination ideal of $\mathcal{I}_{p_i}$, which eliminates x_1 and x_2, but not p_i, can be computed. Hence, in Steps 6 and 7, a formula expressing the faulty parameter, dependent only on the time derivatives of the output, can be obtained. By the same reasoning, in Step 8, a Groebner basis of the elimination ideal of $\mathcal{I}_{p_i}$, which eliminates x_1 and p_i, but not x_2, can be computed. Hence, in Steps 9, 10 and 11, a