European Journal of Control 24 (2015) 2–13

Contents lists available at ScienceDirect

European Journal of Control

journal homepage: www.elsevier.com/locate/ejcon

Distributed randomized algorithms for opinion formation, centrality computation and power systems estimation: A tutorial overview[☆]

Paolo Frasca [a], Hideaki Ishii [b], Chiara Ravazzi [c], Roberto Tempo [d,*]

[a] Department of Applied Mathematics, University of Twente, The Netherlands
[b] Department of Computational Intelligence and Systems Science, Tokyo Institute of Technology, Japan
[c] Department of Electronics and Telecommunications (DET), Politecnico di Torino, Italy
[d] CNR-IEIIT, Politecnico di Torino, Italy

ARTICLE INFO

Article history:
Received 10 December 2014
Received in revised form
10 April 2015
Accepted 17 April 2015
Recommended by A. Astolfi
Available online 25 April 2015

Keywords:
Networked control systems
Distributed randomized algorithms
Opinion formation
Centrality computation
PageRank
Power systems estimation

ABSTRACT

In this tutorial paper, we study three specific applications: opinion formation in social networks, centrality measures in complex networks and estimation problems in large-scale power systems. These applications fall under a general framework which aims at the construction of algorithms for distributed computation over a network. The two key ingredients of *randomization* and *time-averaging* are used, together with a local gossip communication protocol, to obtain convergence of these distributed algorithms to the global synchronous dynamics.

1. Introduction

Over the past decade, the *networks paradigm* has emerged as a central theme in the systems and control community. Networks are now viewed as a research area which includes various applications of significant interest, such as the rapid spread of financial crises and epidemics, aggregation of human behavior and the growth of the Internet, just to name a few topics which are influencing our daily lives. Other scientific communities such as computer science, physics, applied mathematics, biology and social sciences have been involved in the research on "complex networks" since a long time, see e.g. [29]. The tools that are available within systems and control have a distinguishing feature, which is the key synergy of *uncertainty, dynamics and feedback*, which is not available elsewhere. This synergy provides the ultimate objective of this research: to develop feedback mechanisms which deal with dynamic models describing uncertain systems connected through a network with limited communication capacity. Moving on from the control of a given dynamic system, possibly nonlinear and uncertain, to the control of a large number of systems connected through a network is certainly a major step forward and leads to *networked control systems* [5].

More specifically, one of the focal points of the research in recent years has been the study of consensus and coordination of multi-agent systems by means of a graph-theoretic setting which represents a network [16,53]. Significant research has been also performed over the years to develop tools and algorithms for distributed computation and optimization [7,58], distributed estimation based on relative measurements [6,23], clock synchronization of wireless sensor networks [35] and optimal deployment of robotic networks [15].

In this tutorial paper, we study three problems coming from different areas of control engineering: opinion formation, centrality computation and estimation problems in power systems, which are described in Sections 2, 3 and 4, respectively. Our contribution is to recast these applications under a general framework and show some strong similarities in the qualitative picture using tools previously devised in [64]. In particular, in [64] we provide technical results on ergodic properties of distributed dynamics over networks based on a combination of randomization and time-averaging techniques (see also the conference papers [31,65,66]). Moreover, compared to the former paper, the distributed algorithms developed for power systems estimation problems are new.

Randomization has proved to be a useful ingredient, complementary to classical robustness techniques [62], when dealing with control of uncertain systems [69]. In network dynamics, randomization is also quite natural and has the objective to improve the

☆Partial funding for this research is provided by the CNR Joint International Lab COOPS, by Japan Science and Technology Agency under the EMS-CREST program, and by CNR under the Short Term Mobility Program.
* Corresponding author.
E-mail addresses: p.frasca@utwente.nl (P. Frasca), ishii@dis.titech.ac.jp (H. Ishii), chiara.ravazzi@polito.it (C. Ravazzi), roberto.tempo@polito.it (R. Tempo).

http://dx.doi.org/10.1016/j.ejcon.2015.04.002
0947-3580/

overall performance of the system, for example when distributed and asynchronous algorithms should be developed. On the other hand, time-averaging has been widely employed in optimization problems, for example to improve convergence speed of stochastic approximation algorithms [63].

The general framework previously mentioned concentrates on a class of randomized affine dynamics that do not enjoy an equilibrium point, but are stable on average. This stability property guarantees that the dynamics, although affected by persistent random oscillations, possesses an ergodic behavior that can be readily exploited in many network-based dynamics where randomization is (apparently) an obstacle to obtain convergence. In particular, we consider dynamics where nodes interact in randomly chosen pairs, following a so-called gossip protocol [12]. This protocol is particularly appealing, for example, when dealing with sensor networks where battery consumption is a significant concern or with social networks where individuals may discuss various topics in pairs or in small groups.

As a consequence of these observations, the desired convergence property, which holds in expectation, can be recovered by each node of the network through a process of time-averaging. Remarkably, time-averages can be computed *locally* by each node and, in some cases, even without access to a common clock to obtain the *global* synchronous dynamics.

Many network algorithms can be randomized in such a way that the dynamics converges (almost surely) to the same limit of the synchronous dynamics. Nevertheless, examples of randomized algorithms that do not converge have also recently appeared in the literature. Such algorithms require an additional "smoothing" operation in order to converge, and in our framework, this goal is simply obtained by means of time-averaging.

We now briefly describe the three applications studied in this paper; additional details are given subsequently in the general description of these applications.

The first application, see Section 2, arises in social sciences and it is focused on the mechanisms of opinion formation, which plays a significant role in many other areas such as economy, finance, biology and epidemiology. The model we consider is based on the concept of stubborn agents [38], which leads to a disagreement of opinions, and it is an alternative to classical models where the objective is to reach a consensus of opinions between several individuals. Compared to [64], the contribution is a new model of communication between agents and, thus, of opinion evolution. More precisely, we consider symmetric pairwise randomized dynamics. At each time step, a randomly chosen pair of agents update its opinion as a convex combination of its own opinion, the opinion of one of its neighbors, and the so-called prejudice. We show that, even though the resulting dynamics persistently oscillates, its average is a stable opinion profile, which is not a consensus of opinions.

The second application, see Section 3, deals with the computation of centrality measures. In particular, we discuss various measures often used in complex networks [29]: degree centrality, closeness, betweenness and eigenvector centralities, and we compare them by means of a simple illustrative example. We also remark that the eigenvector centrality is closely related to the PageRank algorithm for ranking websites in order of importance [13]. In particular, for PageRank, we study a distributed randomized algorithm and show its convergence properties. This algorithm is based on link randomization, and it is an alternative to other algorithms previously proposed in [42], which are based on node randomization.

The third application, see Section 4, deals with estimation problems in power systems where the large-scale power grids are geographically distributed. The topology of the grid is represented as a network, where each node corresponds to a bus in the grid and the edges connecting the nodes represent the transmission lines. Using a linearized model, a (weighted) least squares approach is generally used to determine the states of the grid including the voltage magnitudes and phase angles at the buses [2,57]. However, this computation may not be practical if it is done centrally for some real-time control and monitoring applications especially when the grid size becomes larger. Hence, distributed computation has become an active area of research [40]. Here, we design new distributed randomized algorithm for estimation based on partitioning of the grid and establish its convergence properties. Simulation results showing the performance of the proposed algorithm are given for the classical IEEE 14-bus test system [24].

Finally, in Section 5, we provide brief conclusions.

1.1. General notation

We begin our work by fixing some notation and by reviewing some definitions of graph theory. We denote the sets of real and nonnegative integer with the symbols $\mathbb{R}$ and $\mathbb{Z}_{\geq 0}$, respectively. The notation $|\cdot|$ is used to indicate either the cardinality of a set or the absolute value of a real number. We denote column vectors with small letters, and matrices with capital letters. The symbol e_i is the vector with the ith entry equal to 1 and all the other elements equal to 0, and we write $\mathbf{1}$ for the vector with all entries equal to 1. A matrix A is row-stochastic (column-stochastic) when its entries are nonnegative and $A\mathbf{1}=\mathbf{1}$ ($A^\top\mathbf{1}=\mathbf{1}$). A matrix A is said to be Schur stable if the absolute value of all its eigenvalues is smaller than 1. Given the sequence of real vectors $\{x(k)\}_{k\in\mathbb{Z}_{\geq 0}}$, we denote its time-average, also known as Cesáro average or Polyak average in some contexts, with

$$\overline{x}(k) := \frac{1}{k}\sum_{\ell=0}^{k-1} x(\ell). \tag{1}$$

A directed graph is a pair $\mathcal{G}=(\mathcal{V},\mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}\subseteq\mathcal{V}\times V$ is the set of edges. We say that $\mathcal{G}=(\mathcal{V},\mathcal{E})$ is an undirected graph if $(i,j)\in\mathcal{E}$ implies that (i,j) is also an edge in $\mathcal{E}$. The set of neighbors of $i\in\mathcal{V}$ is denoted as $\mathcal{N}_i=\{j\in\mathcal{V}:(i,j)\in\mathcal{E}\}$. The degree of a node $i\in\mathcal{V}$ is $|\mathcal{N}_i|$. A path in a graph is a sequence of edges which connect a sequence of vertices. In an undirected graph $\mathcal{G}$ the nodes i and j are said connected if there exists a path from i to j. A graph is said to be connected if every pair of vertices in the graph is connected. To any matrix $P\in\mathbb{R}^{\mathcal{V}\times\mathcal{V}}$ with nonnegative entries, we associate a directed graph $\mathcal{G}_P=(\mathcal{V},\mathcal{E}_P)$ by putting $(i,j)\in\mathcal{E}_P$ if and only if $P_{ij}>0$. The matrix P is said to be adapted to graph $\mathcal{G}$ if $\mathcal{G}_P\subseteq\mathcal{G}$.

2. Opinion formation

The study of opinion formation in social networks has increasingly attracted the attention of the control community in the past decades [8]. The definition of social network is often associated to describe the complex networked systems that play a fundamental role in a number of different fields such as economy, finance, biology, epidemiology, and sociology. In general terms, a social system consists of a large number of agents/individuals whose interactions may induce the emergence of collective behaviors. The global behavior of the network shows often complexity features that can be seen as the result of the addition of the many individual behaviors and the rapid diffusion of information facilitated by the topology of the interconnections.

Researchers devoted their attention on various models for the dynamics of continuous opinions by analytical analysis as well as via numerical simulation. Pioneering works appeared in the early 1950s [34] in order to give theoretical explanations of complex