European Journal of Control 24 (2015) 50–62

Contents lists available at ScienceDirect

European Journal of Control

journal homepage: www.elsevier.com/locate/ejcon

Direct continuous-time approaches to system identification. Overview and benefits for practical applications

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ARTICLE INFO

Article history:
Received 7 January 2015
Received in revised form
17 April 2015
Accepted 17 April 2015
Recommended by A. Astolfi
Available online 25 April 2015

Keywords:
Continuous-time models
System identification
Instrumental variable
Irregularly sampled data

ABSTRACT

This paper discusses the importance and relevance of direct continuous-time system identification and how this relates to the solution for model identification problems in practical applications. It first gives a tutorial introduction to the main aspects of one of the most successful existing approaches for directly identifying continuous-time models of dynamical systems from sampled input–output data. Compared with traditional discrete-time model identification methods, the direct continuous-time approaches have some notable advantages that make them more useful in many practical applications. For instance, continuous-time models are more intuitive to control scientists and engineers in their every-day practice and the related estimation methods are particularly well suited to handle rapidly or irregularly sampled data situations. The second part of the paper describes further recent developments of this reliable estimation technique, including its extension to handle coloured measurement noise situations, time-delay system identification, frequency-domain identification, non-uniformly sampled data, closed-loop and nonlinear model identification. It also discusses the software tools available and illustrates their advantages via simulated and real data examples.

1. Introduction

The growth in the use of parameterised models to accomplish different objectives in the design of industrial control systems has been accompanied by a similar growth in the science of system identification. Today, there is a thriving research community pursuing new developments in system identification that support the use of system models in control design, fault diagnosis and for understanding the nature of the process.

There are three different kinds of parameterised models:

- *grey-box models*, where the model is constructed in continuous-time from basic physical principles and the parameters represent unknown values of the system coefficients that, at least in principle, have a direct physical interpretation. Such models are also known as physically parameterised or tailor-made models;
- *black-box models*, which are families of flexible models of general applicability. The parameters in such models, which can be continuous-time (CT) or discrete-time (DT), have no direct physical interpretation (even though the CT version is closer to the physically parameterised model than the DT version), but are used as vehicles to describe the properties of the input–output relationships of the system. Such models are also known as ready-made models;
- *data-based mechanistic (DBM) models*, which are effectively models identified initially in a black-box, generic model form but only considered credible if they can be interpreted in physically meaningful terms.

In this tutorial paper, we restrict our attention to black-box model identification. The reader is referred, for instance, to [2] and the references therein, for grey-box model identification; and [66] and the references therein, for DBM model identification.

A mainstay of the control system modelling paradigm is continuous-time models because they arise naturally when describing the physical phenomena of systems and processes. These mathematical models of dynamical systems usually involve differential equations that stem from the application of physical and chemical laws.

There are two fundamentally different time-domain approaches to the problem of obtaining a black-box CT model of a natural CT system from its sampled input–output data:

- the *indirect approach*, which involves two steps. First, a DT model for the original CT system is obtained by applying DT

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http://dx.doi.org/10.1016/j.ejcon.2015.04.003
0947-3580/

model estimation methods to the available sampled data; and then the DT model is transformed into the required CT form. This indirect approach has the advantage that it uses well-established DT model identification methods [52,30,67];

- the *direct approach* where a CT model is obtained immediately using CT model identification methods, such as those discussed in this paper. Without relying any longer on analogue computers, the present techniques exploit the power of the digital tools. In this direct approach, the model remains in its original CT form. Many methods were developed in the 1960s and 1970s but they were largely eclipsed at the time by the emphasis on DT model identification. Exhaustive reviews of direct estimation methods can be found in [64,58,59,11,48,45,8].

For many people working in the field of system identification, the choice between the direct and indirect approaches may seem trivial but some recent experience, as illustrated by the numerical example reported below, clearly shows that this is not so.

Almost 15 years ago, with my first PhD student Michel Mensler, we studied the numerous direct continuous-time approaches to system identification and packaged them in the first version of what would become the CONTSID toolbox for Matlab [7]. A few years later, while Ganti Prasada Rao, one of the most active promotors of these direct continuous-time approaches at that time [58,59], was visiting me in Nancy, we suggested together a benchmark system, termed as the Rao–Garnier system by Lennart Ljung [31], which aimed initially at comparing the performance of the traditional DT and CT model identification methods in practice [46,11]. It has been used as a benchmark system in many papers since (see *e.g.* [11,31,47,48,73,3,36]). A typical example of the intriguing results obtained when one identifies DT models by using the popular `ARX`, `N4SID` and `PEM`[1] routines from the Matlab System IDentification (SID) toolbox on one dataset[2] coming from the Rao–Garnier benchmark is shown at the top of Fig. 1 (see for instance [46] or [11] for the full details about the benchmark test).

After some time spent hunting nonexistent bugs in generating the dataset or in the use of the SID routines, I finally understood that the traditional DT identification approaches in their default modes could not deliver good results without some special data pre-treatments which required some expertise from the practioner. These comparative results were first presented at the World IFAC congress in 2002 [46]. The results are intriguing since they are not the results one can expect to have for this kind of, let us say, not too complicated fourth-order *linear* system. I received many messages from system identification experts who were puzzled by the comparative results. The latter were confirmed by Lennart Ljung in an ECC paper the following year in which he exposed the main reasons for the bad results of the traditional DT identification methods [31]. The key problem is that DT ARX models are very biased, which leads to problems for initializations for PEM/OE models both based on ARX/IV4 and subspace techniques. The remedy is to decrease the ARX-bias via low pass data filtering. Note however that the results presented in Fig. 1 have been obtained by using the latest version 2015a of Matlab, and so that the current implementation of the traditional DT algorithms still suffers from the same difficulties.[3] On the contrary, the direct continuous-time approaches seem to be free of these difficulties since, as it can be observed from the bottom of Fig. 1, the magnitude Bode plot of the CT model estimated by the SRIVC method can hardly be distinguished from the true system Bode plot.

My introduction to SRIVC occurred when visiting Peter Young in Lancaster in 2001. Thereafter with the supportive help of good colleagues (Marion Gilson, Torsten Söderström, Liuping Wang and Juan Yuz), I organized several tutorial and invited international conference sessions (mainly at IFAC Symposia on System Identification (SYSID) in Rotterdam (2003), Newcastle (2006), Saint-Malo (2009) and Brussels (2012)) to promote the use of these direct CT schemes to system identification. The outcomes of these endeavours are an edited book in 2008 [8] and two journal special issues, the first for the IET Control Theory & Applications in 2011 [15], the second for International Journal of Control in 2014 [9]. Another outcome of these activities is the development of the CONTSID toolbox for Matlab dedicated to these direct CT approaches [16]. Interestingly, more emphasis has been placed on direct CT estimation schemes in the latest version of the Matlab SID toolbox [33]. Despite these recent activities, it seems that many practitioners appear unaware that such direct CT methods to system identification not only exist, but may be better suited to their modelling problems.

In general, the estimation of the continuous-time model parameters is a non-linear statistical estimation problem that can be solved by using several main approaches, such as the maximum likelihood and prediction error methods or instrumental variable techniques. It is not possible to give a short, comprehensive survey of the field, and many highly relevant estimation methods, results and papers will not be discussed here. Rather, the paper gives my own subjective views. It briefly reviews the main approaches and then concentrates on one of the most efficient direct iterative Instrumental Variable (IV)-based estimation methods for CT linear models known as the SRIVC method. It presents the latest developments for it, including its use for closed-loop and nonlinear model identification. It also describes the software tools available, discusses the advantages of these direct CT methods and presents two examples that demonstrate the practical utility and efficacy of these methods.

2. Identification of continuous-time linear models

A linear time-invariant continuous-time system with input u and output y can be described by[4]

$$y(t) = G(p)u(t) + e(t), \tag{1}$$

where $G(p)$ is the transfer operator model,

$$G(p) = \frac{B(p)}{A(p)} = \frac{b_0 p^m + b_1 p^{m-1} + \cdots + b_m}{p^n + a_1 p^{n-1} + \cdots + a_n}, \quad n \geq m. \tag{2}$$

p is the time-domain differentiation operator and the additive term $e(t)$ represents the measurement error which, to start with, is assumed to be a white noise process. In this situation, the model takes the form of the so-called continuous-time output error (COE) structure so that no explicit noise modelling is necessary, except in relation to the estimation of the variance of the white noise process. The white noise assumption is not restrictive and is required for deriving the optimal solution in the proposed algorithm. It has to be remembered that in practice, the model mismatch is not due to measurement errors but to the infinite dimensional, nonlinear, time-varying nature of the phenomenon that produces the data and the finite dimensional, linear,

[1] Actually the `OE` routine here since the additive measurement noise is white for the dataset.

[2] The simulation conditions correspond to trial9 of the extensive Monte Carlo simulation analysis presented in [11]. The excitation signal is a PRBS, the sampling period is set to 10 ms, the additive measurement noise is white and the signal-to-noise ratio equals 10 dB.

[3] It has to be mentioned that the `OE` routine can deliver good estimation results depending on the realization of the noise.

[4] A time-delay on the system input is not considered for simplicity here but is easy to accommodate.