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Unknown-input observability with an application to prognostics for Waste Water Treatment Plants

Frédéric Lafont [a,b,*], Nathalie Pessel [a,b], Jean-François Balmat [a,b], Jean-Paul Gauthier [a,b,c]

[a] Aix Marseille Université, CNRS, ENSAM, LSIS, UMR 7296, 13397 Marseille, France
[b] Université de Toulon, CNRS, LSIS, UMR 7296, 83957 La Garde, France
[c] INRIA Project GECO, France

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ABSTRACT

This paper is devoted to the problem of model-based prognostics for a Waste Water Treatment Plant (WWTP). Our aim is to predict degradation of certain parameters in the process, in order to anticipate malfunctions and to schedule maintenance. It turns out that a WWTP, together with the possible malfunction, has a specific structure: mostly, the malfunction appears in the model as an unknown input function. The process is observable whatever this unknown input is, and the unknown input can itself be identified through the observations. Due to this property, our method does not require any assumption of the type "slow dynamics degradation", as is usually assumed in ordinary prognostic methods. Our system being unknown-input observable, standard observer-based methods are enough to solve prognostic problems. Simulation results are shown for a typical WWTP.

1. Introduction

The prognostic concept for the monitoring and supervision of technological systems has been under development for several years. The main purpose is the estimation of the remaining useful life (RUL) and/or the risk of one or more existing or future failure modes [8,28,16].

Two current prognostic approaches can be considered [8]:

- *Data-driven prognostic*: Data-driven approaches use real data to detect the degradation of components and to predict the global behavior of a system. This type splits up into two categories: Artificial Intelligence (AI) techniques (neural networks, fuzzy systems, decision trees, etc.) and statistical techniques (multivariate statistical methods, linear and quadratic discriminators, partial least squares, etc.). The strong point of data-driven techniques is their ability to transform high-dimensional noisy data into lower dimensional information for diagnosis/prognostic decisions.
  - AI techniques have been increasingly applied in the field of prediction and have shown improved performances over conventional approaches. However, in practice, it is not easy to apply AI techniques due to the lack of effective procedures to collect data source and specific knowledge.
  - Statistical techniques require quantitative data measurements. These informations are combined to provide a stochastic estimation of the future state [7,12,24].
- *Model-based prognostic*: Model-based methods assume that reasonably accurate mathematical models of either the system or the degradation processes or both are available. For instance, a model of the degradation may be known, up to a certain number of parameters. If these parameters are unknown, they can be added to the model as constant state variables to be estimated. In [8], the authors indicate that, similar to diagnosis, these methods often use residuals which are the differences between the measurements from the system and the outputs of the mathematical model. These residuals are large in the presence of malfunctions and small in the presence of normal disturbances (noise and modeling errors). Several techniques are proposed in the literature to generate residuals: parity space, parameter estimations, observers, bond graphs, etc. The main advantage of model-based approaches is their ability to incorporate physical understanding of the monitored system and/or the malfunctions.

  Regarding models of specific failure mechanisms, we can quote physics-based fatigue models, which have been extensively used to represent the initiation and propagation of structural anomalies. The authors [6,13,21] developed a dynamical-system approach to estimate the damage evolution.

* Corresponding author.
E-mail address: lafont@univ-tln.fr (F. Lafont).

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Often, a slow-fast assumption is made: the dynamics of the degradation is slow w.r.t. the proper dynamics of the system. This is the case in the above series of papers. For instance, this slow-fast assumption could allow the use of Tikhonov-like arguments [18,27].

The slow-fast assumption leads to a set of equations as follows:

$$\dot{x} = f(x, \theta(\phi), u), \quad x(t_0) = x_0 \tag{1.a}$$

$$\dot{\phi} = \varepsilon g(x, \phi), \quad \phi(t_0) = \phi_0 \tag{1.b}$$

$$y = h(x, \phi, u) \tag{1.c}$$

where $x \in \mathbb{R}^n$ is the set of n state variables associated with the fast dynamic behavior of the system, $\phi \in \mathbb{R}^k$ is the set of k slow dynamic variables related to system damage, $\theta \in \mathbb{R}^r$ is the parameter vector assumed to be a function of ϕ, $u \in \mathbb{R}^m$ is the input vector (m inputs), the rate constant $0 < \varepsilon \ll 1$ defines the time-scale separation between fast dynamic and slow drift, $y \in \mathbb{R}^p$ is the output vector (p outputs). The functions f, g, and h are smooth. In [18,27], the authors consider that the damage value ϕ can be represented by a model with a known structure (polynomial form, for instance) in order to estimate the RUL.

Another approach, which is often combined with the slow-fast assumption, consists of considering the state ϕ of the degradation as an unknown input. The estimation problem is then related to the difficult question of observability for unknown inputs [2–4].

The study presented here is a contribution to prognostic problems without any assumption of the type "slow dynamics degradation" thanks to the structure of the system. The interest of this method is to trace the degradation with no a priori knowledge (in this study, a degradation model is only used in the simulations, but it is not used for the proposed prognostic procedure). Our methodology is no longer based upon a difference between fast and slow dynamics. The main objective remains the prediction of the RUL, which is an important factor for improving process safety and scheduling maintenance.

The paper is organized as follows. Section 2 is devoted to our methodology developed for systems with the degradation affecting a single output of the system only. This structure is in fact inherited from our application presented in Section 3. The Waste Water Treatment Plant (WWTP). We use the equations of the process in a simplified form, which is enough for our prognostic purpose [5]. Regarding the WWTP, only the two main types of degradation that are likely to occur have been taken into account:

- A tank leakage phenomenon.
  This case can be treated using the equations of the mass balance of the process only.
- A loss of efficiency of the motor providing tank oxygenation.
  This case is solved using the new methodology proposed here.

Section 3.5 shows simulation results from simulated noisy data.

At this point, we must emphasize the following key fact. Our methodology here is specially simple inside the considered class of systems with the degradation affecting a single output variable. However, it follows from [3,4] that in the class of systems with 3-outputs, zero-known-input, one-unknown-input, observability whatever the unknown-input is a generic property.

Remark 1. In the case of WWTP, we may consider that we are in the zero-known-input case: actually, the control known-input (the aeration of the WWTP) is either maximum or zero. Moreover, when it is zero, the system is unobservable, and nothing better can be done but simple prediction.

2. Our methodology

Our methodology does not require any assumption of the type "slow dynamics degradation". It is applicable within a certain class of systems only, whose structure is inherited from the WWTP model (itself described in a simplified form in Section 3.2).

2.1. General idea

To be perfectly clear, let us explain our view of prognostics. First of all, certain characteristic inside the model is subject to possible degradation. It is natural to consider it as an unknown input for the system.

It is also natural to assume that the degradation is a dynamic phenomenon, modeled by a certain (possibly slow) differential equation, via a finite number of parameters. However, this latter (parametric) approach may be very dangerous, even in the linear context. Actually, let us focus on the linear case (general nonlinear situation is even much worse).

Consider a linear system,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2}$$

Assume for simplicity that the (single) unknown input is $u(\cdot)$. Then, the linear mapping $P : (x_0, u(\cdot)) \to y(\cdot)$ writes

$$y(t) = Ce^{At}x_0 + C\int_0^t e^{A(t-\sigma)}Bu(\sigma)\, d\sigma = P(x_0, u(\cdot))(t) \tag{3}$$

It turns out that, in general, this mapping P has a nonzero kernel. Therefore, generically, the (linear) system is not unknown-input observable. However, choosing some model for the degradation restricts the domain of the linear map P, and may make it injective over the restricted domain. For instance, generically, any polynomial parametrization of any degree of the unknown input u makes the mapping P injective. Therefore, this approach should be considered with suspicion.

This problem disappears inside the class of systems such that the map P is injective. These systems have been fully characterized in the single output and 2-output cases in [2–4], in the general nonlinear context. In these cases, this property of observability for unknown-input is highly non-generic. However, for the WWTP, we are in the 3-output case in which unknown-input-observability becomes generic [3,4]. Moreover, the WWTP has the particularity that the degradation affects a single output variable. In this case, it is completely obvious to check observability for unknown-inputs within this class.

This means that the degradation can be reconstructed independent of any model. This degradation being reconstructed on-line, it is not always possible to infer on its future behavior, which could be totally stochastic. However, if the observed degradation is regular enough, certain extrapolation may reasonably be performed.

2.2. Class (C) of systems under consideration

Systems (Σ) under consideration are of the form:

$$\frac{dz}{dt} = s(w, z) \tag{4a}$$

$$y_0 = w \tag{4b}$$

$$y_i = q_i(w, z), \quad i = 1, \ldots, p \tag{4c}$$