European Journal of Control 19 (2013) 300–312

Contents lists available at SciVerse ScienceDirect

European Journal of Control

journal homepage: www.elsevier.com/locate/ejcon

Selective model inversion and adaptive disturbance observer for time-varying vibration rejection on an active-suspension benchmark

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ARTICLE INFO

Article history:
Received 4 January 2013
Accepted 30 April 2013
Recommended by I.D. Landau/D.W. Clarke
Available online 10 May 2013

Keywords:
Adaptive regulation
Loop shaping
Inverse control
Vibration rejection
Active suspension

ABSTRACT

This paper presents an adaptive control scheme for identifying and rejecting unknown and/or time-varying narrow-band vibrations. We discuss an idea of safely and adaptively inverting a (possibly non-minimum phase) plant dynamics at selected frequency regions, so that structured disturbances (especially vibrations) can be estimated and canceled from the control perspective. By taking advantage of the disturbance model in the design of special infinite-impulse-response (IIR) filters, we can reduce the adaptation to identify the minimum amount of parameters, achieve accurate parameter estimation under noisy environments, and flexibly reject the narrow-band disturbances with clear tuning intuitions. Evaluation of the algorithm is performed via simulation and experiments on an active-suspension benchmark.

1. Introduction

The rejection of single and multiple narrow-band disturbances is a fundamental problem in many mechanical systems that involve periodic motions. For example, the shaking mechanism in active suspensions [13], the rotating disks in hard disk drives [2], and the cooling fans for computer products [11], all generate vibrations that are composed of sinusoidal components in nature. Challenges of the problem are that we often do not have accurate knowledge of the disturbance frequencies, and that in many applications the disturbance characteristics may even change w.r.t. time and/or among different products (e.g., in the hard disk drive industry [7]).

In various situations, hardware limitations or excessive costs make it infeasible to re-design the hardware for reducing these disturbances, and it is only possible to address the problem from the control-engineering approach. As narrow-band disturbances are composed of sinusoidal signal components, controllers can be customized to incorporate the disturbance structure to asymptotically reject the vibrations. This internal-model-principle [6] based perspective has been investigated in feedback control algorithms in [27,10,1,11,13], among which Refs. [10,27] used state-space designs, and Refs. [1,11,13] applied Youla parameterization, a.k.a. all stabilizing controllers, with a finite impulse response (FIR) adaptive Q filter. Alternatively, the disturbance frequency can be firstly estimated and then applied for controller design. This indirect-adaptive-control perspective has been considered in [12,9,3]. Indeed, frequency identification of narrow-band signals is a problem that receives great research attention itself. Among the related literature we can find: (i) methods using nonparametric spectrum estimation or eigen analysis (subspace methods) [22,23]; (ii) online adaptive identification approaches [19,20,8,18,25,17,16]. Spectrum estimation and eigen analysis in general require more expensive computation within the sampling interval, and have lower resolutions for non-stationary processes. Among references in group (ii), for the identification of n frequency components, adaptive notch filters in the orders of $5n-1$ [17], $2n+6$ [16], $3n$ [20,8,18], and $2n$ [19], have been discussed.

In this paper we discuss a new adaptive incorporation of the internal model principle for asymptotic rejection of narrow-band disturbances. Different from the FIR-based adaptive algorithms, we construct the controllers and adaptation with infinite impulse response (IIR) filters and inverse system models. Applying these considerations we are able to obtain a structured controller parameterization that requires the minimum number of adaptation parameters. An additional consideration is that adaptation on IIR structures enables direct adaptive control with adaptation algorithms that use the parallel predictor, which is essential for accurate parameter convergence under noisy environments [15]. The importance of this aspect can also be seen from the aforementioned literature on frequency identification. Finally, with the inverse-model based design, the internal signals in the proposed algorithm have clear physical meanings. The controller structure extends the idea in [2]. The main results of this paper, i.e., the design of inverse models, the derivation of the cascaded IIR filters,

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http://dx.doi.org/10.1016/j.ejcon.2013.04.002

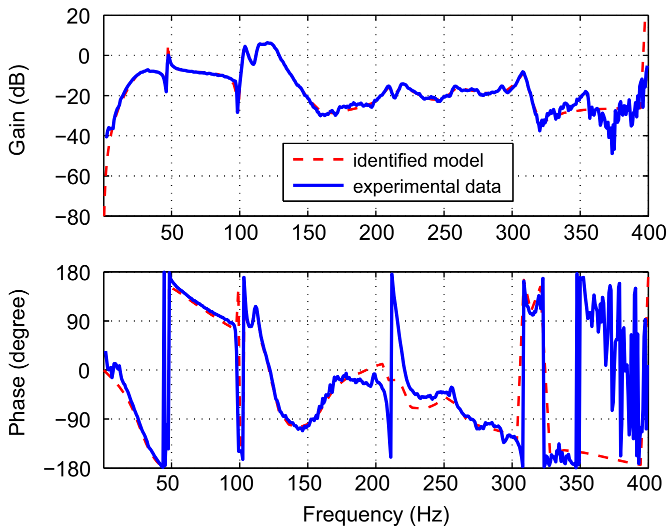

Fig. 1. Frequency response of the plant.

and the adaptation algorithm for time-varying disturbance rejection, however, are all newly developed. An additional contribution is the application to a new class of systems that has characteristics quite different from the hard disk drive in [2]. A short version of the paper appeared in [4]. This paper is a substantially modified version with proofs and derivations of equations, extended analysis of the algorithms (especially the adaptive-control part), and implementation details as well as the full simulation and experimental results.

The algorithm is evaluated on an active suspension system that has been described in [14]. Fig. 1 presents the frequency response of the plant. It can be observed that the system has a group of resonant and anti-resonant modes, especially at around 50 Hz and 100 Hz. Additionally, the system is open-loop stable but has multiple lightly damped mid-frequency zeros and high-frequency nonminimum-phase zeros. These characteristics place additional challenges not just for adaptive disturbance rejection, but also for general feedback control [24].

2. Selective model inversion

Fig. 2 shows the proposed control scheme. We have the following relevant signals and transfer functions:

- $P(z^{-1})$ and $\hat{P}(z^{-1})$: the plant and its identified model. They are open-loop stable in the benchmark;
- $C(z^{-1})$: the baseline controller designed to provide a robustly stable closed loop. For controlling a stable plant, we assume that $C(z^{-1})$ is also stable;
- $d(k)$ and $\hat{d}(k)$: the actual (unmeasurable) disturbance and its online estimate. To see this, note that

$$\hat{d}(k) = P(q^{-1})u(k) + d(k) - \hat{P}(q^{-1})u(k) \approx d(k),$$

where q^{-1} is the one-step-delay operator (in this paper, $P(z^{-1})$, $P(q^{-1})$, and $P(e^{-j\omega})$ are used to denote respectively the transfer function, the pulse transfer function, and the frequency response of $P(z^{-1})$). $\hat{d}(k)$ is quite accurate as $\hat{P}(z^{-1})$ is identified quite accurately in the benchmark. The minimum requirement for $\hat{P}(z^{-1})$ is that it should be accurate within the frequency region where disturbance occurs. The noise due to model mismatch, if any, can be later reduced by filtering in the adaptation scheme;
- $y(k)$: the measured residual errors;

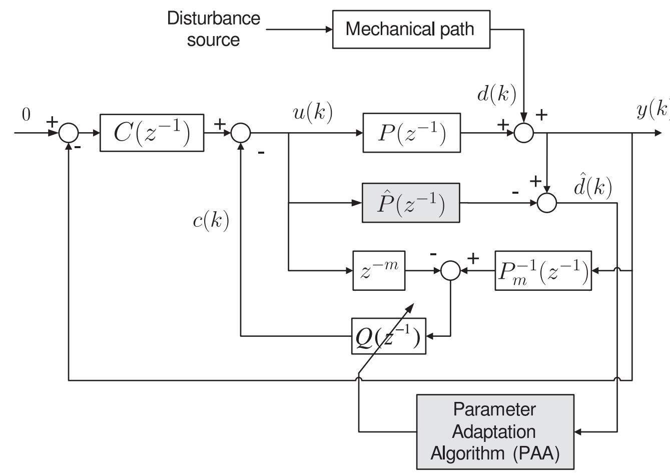

Fig. 2. Structure of the proposed control scheme.

- $P_m^{-1}(z^{-1})$ and z^{-m}: these are constructed such that:
  - (i) $P_m(z^{-1})$ has a relative degree of zero (and hence $P_m^{-1}(z^{-1})$ is realizable);
  - (ii) $P_m^{-1}(z^{-1})$ is stable;
  - (iii) within the frequency range of the possible disturbances, $P(z^{-1})|_{z=e^{j\omega}} \approx z^{-m}P_m(z^{-1})|_{z=e^{j\omega}}$, namely, $P_m^{-1}(z^{-1})$ is a nominal inverse model (without delays) of $P(z^{-1})$. Indeed in Fig. 1, the dashed line depicts the frequency response of $z^{-m}P_m(z^{-1})$, which matches well with the experimental frequency response up to around 350 Hz;
- Parameter adaptation algorithm: provides online information of the characteristics of $\hat{d}(k)$;
- $c(k)$: the compensation signal to asymptotically reject the narrow-band disturbance in $d(k)$.

Ignoring first the shaded blocks (about parameter adaptation) in Fig. 2, we have

$$y(k) = d(k) + P(q^{-1})u(k)$$

$$u(k) = -C(q^{-1})y(k) - c(k)$$

$$c(k) = Q(q^{-1})[P_m^{-1}(q^{-1})y(k) - q^{-m}u(k)]$$

from which we can derive the sensitivity function, namely, the transfer function from $d(k)$ to $y(k)$

$$S(z^{-1}) = G_{d2y}(z^{-1}) = (1 - z^{-m}Q(z^{-1}))/X(z^{-1}) \tag{1}$$

$$X(z^{-1}) = 1 + P(z^{-1})C(z^{-1}) + Q(z^{-1})(P_m^{-1}(z^{-1})P(z^{-1}) - z^{-m}) \tag{2}$$

From the frequency-response perspective (replace z^{-1} with $e^{-j\omega}$), if $P(e^{-j\omega}) = e^{-jm\omega}P_m(e^{-j\omega})$ in (1) and (2), then the last term in $X(e^{-j\omega})$ vanishes and

$$S(e^{-j\omega}) = (1 - e^{-jm\omega}Q(e^{-j\omega}))/(1 + P(e^{-j\omega})C(e^{-j\omega})). \tag{3}$$

If we design a Q-filter $Q(z^{-1})$ with its frequency response as shown in Fig. 3, then $1 - e^{-mj\omega}Q(e^{-j\omega})$, and thus $G_{d2y}(e^{-j\omega})$ in (1), will become zero at the center frequencies of $Q(z^{-1})$, namely, disturbances occurring at these frequencies will be strongly attenuated. If vibrations occur exactly at 60 Hz and 90 Hz, $Q(z^{-1})$ in Fig. 3 will filter out all other frequency components such that its output $c(k)$ consists of signals only at the disturbance frequencies. More specifically, $c(k)$ is actually an estimated version of $P^{-1}(q^{-1})d(k)$ if $d(k)$ contains just narrow-band vibrations. To see this, notice in Fig. 2, that the control signal $u(k)$ flows through two paths to reach the summing junction before $Q(z^{-1})$: one from the plant $P(z^{-1})$ to the inverse $P_m^{-1}(z^{-1})$, and the other through z^{-m}. Hence the effect