European Journal of Control 19 (2013) 326–336

Contents lists available at SciVerse ScienceDirect

European Journal of Control

journal homepage: www.elsevier.com/locate/ejcon

Direct adaptive rejection of unknown time-varying narrow band disturbances applied to a benchmark problem

Abraham Castellanos Silva [a], Ioan Doré Landau [a,*], Tudor-Bogdan Airimiţoaie [a,b]

[a] Control system department of GIPSA-lab, St. Martin d'Héres, 38402, France
[b] Faculty of Automatic Control and Computers, University "Politehnica" of Bucharest, Bucharest 060042, Romania

ARTICLE INFO

Article history:
Received 27 December 2012
Accepted 5 May 2013
Recommended by A. Karimi/D.W. Clarke
Available online 9 May 2013

Keywords:
Adaptive regulation
Active vibration control
Inertial actuators
Multiple narrow band disturbances
Youla–Kučera parametrization
Internal model principle

ABSTRACT

The paper presents a direct adaptive algorithm for the rejection of unknown time-varying narrow band disturbances, applied to an adaptive regulation benchmark. The objective is to minimize the residual force by applying an appropriate control signal on the inertial actuator in the presence of multiple and/or unknown time-varying disturbances. The direct adaptive control algorithm is based on the internal model principle (IMP) and uses the Youla–Kučera (YK) parametrization. A direct feedback adaptive regulation is proposed and evaluated both in simulation and real-time. The robustness is improved by shaping the sensitivity functions of the system through band stop filters (BSF).

1. Introduction

The problem posed by this benchmark [15] is the attenuation (rejection) of multiple narrow band disturbances of unknown and time-varying frequencies without measuring them. The energy of these disturbances (or vibrations) is concentrated in narrow bands around some unknown frequencies and could be modelled as a white noise or a Dirac impulse passed through a *model of the disturbance*. While, in general, one can assume a certain structure for such *model of disturbance*, its parameters are unknown and may be time-varying. The need of an adaptive approach arises.

A *feedback* approach can provide disturbance rejection (at least asymptotically), using the measurement of the residual force (acceleration) as in [1,2,17]. In this benchmark as well as in many other applications one can consider that a model of the compensator system (which includes the actuator providing disturbance compensation capabilities) is available (obtained in general by system identification). This model is in general time invariant even if one has to consider that uncertainties in the model may be present in certain frequency regions. The approach which is proposed for solving the benchmark problem belongs to the class of solutions using the internal model principle (IMP) [1,2,5,9–12,17,23,24]. Other related references are [6,7,4,20,21,8]. Since the model of the disturbance is considered unknown, an adaptive configuration has to be considered. *Direct* or *indirect* adaptive regulation schemes can be built.

Through the use of the Youla–Kučera (YK) parametrization of the controller and the Internal Model Principle (IMP) a direct adaptive regulation scheme can be built. Direct adaptive schemes are simpler and require less computational time than indirect schemes. They provide in general excellent adaptation transients and stability proofs are available for realistic operational conditions [17]. This approach has been successfully used in a number of applications [17,16,13], and therefore has been considered to be applied to the benchmark.

The YK parametrization (known also as the Q-parametrization) allows to insert and adjust the internal model of the disturbance into the controller by adjusting the parameters of the polynomial $\hat{Q}(z^{-1})$ (see Fig. 1). This is done without recomputing the *central controller* ($R_0(z^{-1})$ and $S_0(z^{-1})$ in Fig. 1 remain unchanged). The number of parameters to be directly adapted is roughly equal to the number of parameters in the denominator of the disturbance model. This means that the size of the adaptation algorithm will depend upon the complexity of the disturbance model and not upon the complexity of the plant model. It is also important to remind that feedback compensation of the disturbances can be done only in the frequencies region where the plant (the compensator system) has enough gain [16].

* Corresponding author.
E-mail addresses: abraham.castellanos-silva@gipsa-lab.grenoble-inp.fr (A. Castellanos Silva), ioan-dore.landau@gipsa-lab.grenoble-inp.fr (I.D. Landau), tudor-bogdan.airimitoaie@gipsa-lab.grenoble-inp.fr (T.-B. Airimiţoaie).

0947-3580/$ - see front matter © 2013 European Control Association. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ejcon.2013.05.012

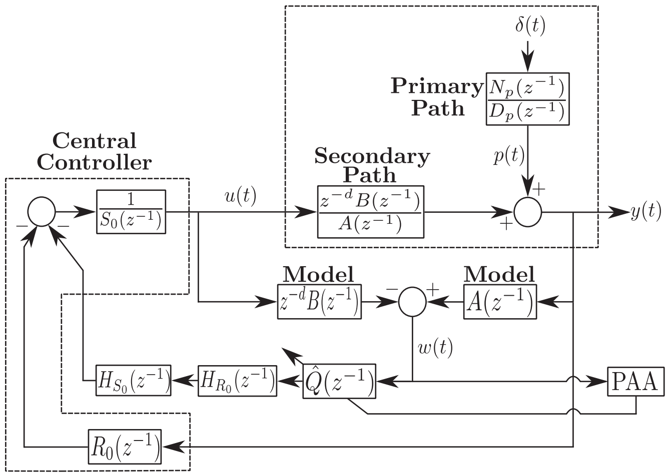

Fig. 1. Direct adaptive regulation scheme for rejection of unknown disturbances.

The major problem encountered with this approach is the design of the central controller such that for any internal model of the disturbances (i.e. for all possible values of the frequencies of the disturbances) within the range of frequencies considered, good robustness of the system (modulus margin, delay margin, low magnitude of input sensitivity function outside the region of compensation) is assured as well as a low amplification at other frequencies than those of the disturbances (one need to get a flat "water bed" effect). The problem becomes even more difficult when there are several narrow band disturbances to be compensated simultaneously which is the case for levels 2 and 3 of the benchmark. One of the main original contributions of this paper is a methodology for the design of the central controller for the case of multiple narrow band disturbances in order to allow satisfaction of benchmark specifications in adaptive operation. It is important to underline that even in the linear case with constant parameters, the design of the central controller is difficult in the case of the benchmark as a consequence of the presence of two pairs of very low damped zeros in the plant model very near to the border of the frequency region where disturbance compensation has to be achieved.

The paper is organized as follows. Section 2 presents the general plant and controller structure in the context of the YK parametrization. The direct adaptive algorithm is presented in Section 3. Section 4 discusses the design of the central controller. Simulation results are presented in Section 5, while experimental results for this methodology are given in Section 6. Concluding remarks are presented in Section 7.

2. Plant representation and controller structure

The structure of the LTI discrete time model of the plant (the compensator system), also called *secondary path*, used for controller design is

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})}, \tag{1}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A}, \tag{2}$$

$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} = z^{-1}B^*, \tag{3}$$

$$B^* = b_1 + \cdots + b_{n_B} z^{-n_B+1}, \tag{4}$$

and d is the plant pure time delay in number of sampling periods.[1]

Without considering a reference signal, the output of the plant $y(t)$ and the input $u(t)$ may be written as (see Fig. 1)

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})} \cdot u(t) + p(t), \tag{5}$$

$$S(q^{-1}) \cdot u(t) = -R(q^{-1}) \cdot y(t). \tag{6}$$

In (5), $p(t)$ is the effect of the disturbances on the measured output[2] and $R_0(z^{-1})$, $S_0(z^{-1})$ are polynomials in z^{-1} having the following expressions[3]:

$$S_0 = 1 + s_1^0 z^{-1} + \ldots + s_{n_{S_0}}^0 z^{-n_{S_0}} = S'_0(z^{-1}) \cdot H_{S_0}(z^{-1}), \tag{7}$$

$$R_0 = r_0 + r_1^0 z^{-1} + \ldots + r_{n_{R_0}}^0 z^{-n_{R_0}} = R'_0(z^{-1}) \cdot H_{R_0}(z^{-1}), \tag{8}$$

where $H_{S_0}(q^{-1})$ and $H_{R_0}(q^{-1})$ represent pre-specified parts of the controller (used for example to incorporate the internal model of a disturbance or to open the loop at certain frequencies) and $S'_0(q^{-1})$ and $R'_0(q^{-1})$ are computed.

We define the output sensitivity function (the transfer function between the disturbance $p(t)$ and the output of the system $y(t)$) as

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{P(z^{-1})} \tag{9}$$

and the input sensitivity function (the transfer function between the disturbance $p(t)$ and the control input $u(t)$) as

$$S_{up}(z^{-1}) = -\frac{A(z^{-1})R(z^{-1})}{P(z^{-1})}, \tag{10}$$

where

$$P(z^{-1}) = A(z^{-1})S_0(z^{-1}) + z^{-d}B(z^{-1})R_0(z^{-1}), \tag{11}$$

the characteristic polynomial, specifies the desired closed loop poles of the system[4] (see also [19]). It is important to remark that one should only reject disturbances located in frequency regions where the plant model has enough gain. This can be seen by looking at Eq. (9) and noticing that perfect rejection at a certain frequency, ω_0, is obtained *iff* $S(e^{-j\omega_0}) = 0$. But from Eq. (10) one can see that the modulus of the input sensitivity function at this frequency is given by

$$|S_{up}(e^{-j\omega_0})| = \left|\frac{A(e^{-j\omega_0})}{B(e^{-j\omega_0})}\right|.$$

The modulus of the input sensitivity function at this frequency is equal to the inverse of the plant gain at this frequency. Therefore, low plant gain will imply that the robustness *vs* additive plant model uncertainties is reduced and the stress on the actuator will become important. Furthermore, it can be observed that serious problems will occur if $B(z^{-1})$ has complex zeros close to the unit circle at frequencies where an important attenuation of disturbances is introduced. It is mandatory to avoid attenuation of disturbances at these frequencies [16].

In this paper, the Youla–Kučera parametrization [3,23] is used. Supposing a finite impulse response (FIR) representation of the

[1] The complex variable z^{-1} will be used to characterize the system's behaviour in the frequency domain and the delay operator q^{-1} will be used for the time domain analysis.

[2] The disturbance passes through a so called *primary path* which is represented in this figure, and $p(t)$ is its output.

[3] The argument (z^{-1}) will be omitted in some of the following equations to make them more compact.

[4] It is assumed that a reliable model identification is achieved and therefore the estimated model is assumed to be equal to the true model.