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Practical stability of switched systems without a common equilibria and governed by a time-dependent switching signal

Roman Kuiava [a,*], Rodrigo A. Ramos [b], Hemanshu R. Pota [c], Luis F.C. Alberto [b]

[a] Federal University of Parana (UFPR), Polytechnic Center, Department of Electrical Engineering, Rua Cel. Francisco Heraclito dos Santos, 100, Jardim das Americas, 81531-980 Curitiba, PR, Brazil

[b] University of Sao Paulo (USP), Engineering School of Sao Carlos (EESC), Department of Electrical Engineering, Av. Trabalhador Saocarlense, 400, Centro, 13566-590 Sao Carlos, SP, Brazil

[c] University of New South Wales at Australian Defence Force Academy (UNSW@ADFA), School of Engineering and Information Technology, Northcott Drive, Canberra, ACT 2600, Australia

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ABSTRACT

In this paper, the problem of practical stability of some classes of continuous-time switched systems is studied. The main results of this paper include some sufficient conditions concerning practical stability of continuous-time switched nonlinear systems without a common equilibria for all subsystems. In this class of switched systems, the equilibrium point varies discontinuously according to a time-dependent switching signal. So, stability with respect to a set, rather than a particular point, is discussed. Using this preliminary result, we present sufficient conditions in the form of linear matrix inequalities (LMIs) for practical stability of a particular class of switched systems without common equilibria: the switched affine systems. An illustrative example in the power system stability area is presented to show the validity of the results.

1. Introduction

Continuous-time switched systems are typically represented by equations in the general form

$$\dot{x}(t)=f_{\sigma(\cdot)}(x(t)),\quad x(t_0)=x_0, \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, t_0 is the initial time and σ is a piecewise constant function called *switching signal*. In this paper, we consider the switching signal is time-dependent, so σ is a function of time defined as $\sigma(t):I\to S$, where $I=[t_0,t_f)$ and t_f is a finite constant. Also, $S=\{1,\dots,N\}$ is a finite set of positive integers. Hence, given a set of subsystems $\{f_1,\dots,f_N\}$, the switching signal is such that $f_{\sigma(t)}\in\{f_1,\dots,f_N\}$, for each $t\in I$. This obviously imposes a discontinuity on $f_{\sigma(t)}$ since this vector jumps instantaneously from f_i to f_j for some $i\neq j$, $i,j\in S$, once switching occurs. The instants of time at which $f_{\sigma(t)}$ is discontinuous, *i.e.*, $t_1,t_2,\dots,t_k,\dots\in I$ $(t_0<t_1<t_2<\cdots<t_k<\cdots)$, are called *switching times*.

The analysis of the dynamic behavior of continuous-time switched systems has been addressed by several authors, whose studies are typically focused on stability [14,12,19,17], controllability, observability [2,15] and the design of controllers with guaranteed stability and performance [8,6,13]. In general, the main results in these areas assume that all subsystems of (1) share a common equilibrium (typically the origin $x=0$, *i.e.*, $f_i(0)=0$ for all $i=1,\dots,N$), and hence the stability of (1) is actually the stability of this common equilibrium [6,5,10,23].

For this class of switched systems, some results of asymptotic stability of the origin can be established from direct methods, making use of an auxiliary scalar-valued Lyapunov function. By using direct methods, the global asymptotic stability of the origin is guaranteed if there exists a common, continuously differentiable, positive-definite real-valued Lyapunov function $v(x(t))$ for all subsystems $\{f_1,\dots,f_N\}$, such that

$$\frac{\partial v}{\partial x}f_i<-\lambda_i v,\quad \lambda_i>0,\quad \forall i=1,\dots,N,\ \ x\neq 0. \tag{2}$$

The existence of a common Lyapunov function for all subsystems $\{f_1,\dots,f_N\}$ ensures that the equilibrium point $x=0$ is globally asymptotically stable for any arbitrary switching signal [6]. Unfortunately, it is well known that, depending on the system behavior and its dimensions, the search for a feasible solution for inequality (2) can be a very difficult task or a solution may not even exist, once that some switched systems may be stable only under restricted switching signals. To study the stability of switched systems under restricted switching rules, the necessity of a common Lyapunov function for all subsystems can be replaced by the existence of a family of continuously differentiable, radially unbounded, positive-definite real-valued

* Corresponding author.
E-mail addresses: kuiava@eletrica.ufpr.br (R. Kuiava), ramos@sc.usp.br (R.A. Ramos), h.pota@adfa.edu.au (H.R. Pota), lfcalberto@usp.br (L.F.C. Alberto).

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Lyapunov functions $\{V_1(x(t)),\dots,V_N(x(t))\}$ such that

$$\frac{\partial V_i}{\partial x} f_i < -\lambda_i V_i, \quad \lambda_i > 0, \quad \forall i = 1,\dots,N, \quad x \neq 0. \tag{3}$$

and,

$$V_{i_{k+1}}(x(t_k)) \le V_{i_k}(x(t_k)), \tag{4}$$

for every switching time $t_k \in I$ at which σ switches from i_k to i_{k+1}, where $i_k,\ i_{k+1} \in S$, $i_k \neq i_{k+1}$ [14,19]. In spite of condition (2), this approach allows to the Lyapunov function $v(x(t)) = V_{\sigma(t)}(x(t))$ to have certain discontinuities (at the switching times), which turns out to be attractive for stability analysis of switched systems. The asymptotic stability of the origin is then verified when the subsystems $\{f_1,\dots,f_N\}$ are individually stable (condition (3)) and $v(x(t))$ is uniformly decreasing for all $t \in I$ (condition (4)).

A less conservative result (in comparison with both conditions (2) and the pair (3) and (4)) is given in Geromel and Colaneri [12], Colaneri et al. [6], where the above nonincreasing condition on the Lyapunov functions is relaxed. Basically, it is replaced by a weaker condition which imposes that the sequence $v(x(t_k))$, $t_0, t_1, \dots, t_k, \dots \in I$ must converge uniformly to zero, where $v(x(t_k)) = V_{i_k}(x(t_k))$ when σ switches to mode i_k at the switching time t_k. In other words, it is required that

$$V_{i_{k+1}}(x(t_{k+1})) \le V_{i_k}(x(t_k)), \quad i_k, i_{k+1} \in S, \quad i_k \neq i_{k+1}, \tag{5}$$

for all successive switching times k and $k+1$, where $\sigma(t) = i_k \in S$, $\forall t \in [t_k, t_{k+1})$.

On the other hand, when there is not a common equilibria for all subsystems of (1), the stability analysis becomes more complicated. An interesting approach concerning stability and stabilization of switched systems without a common equilibria was investigated in Zhai and Mitchel [28], Zhai and Michel [27], Xu and Zhai [24], Xu et al. [25] in terms of *practical stability*. Generally speaking, this concept deals with two sets $\Omega_1 \subset \mathbb{R}^n$ and $\Omega_2 \subset \mathbb{R}^n$ satisfying $\Omega_1 \subset \Omega_2$, which are specified for the initial state and the entire system state, respectively. These two sets do not have to include the origin and they can be specified in terms of physical limitations of the system variables. The practical stability requires that if the initial state is in Ω_1, then the system state should stay in Ω_2 for all $t \in I$ [27].

Hence, unlike the classical stability definition which is based on the existence of Ω_1 for any Ω_2, here both of the sets Ω_1 and Ω_2 are fixed and so they do not vary. Stability with respect to a set, rather than a particular point, is then the basis of the practical stability concept.

Hence, motivated by the studies of practical stability addressed in Zhai and Michel [27], Xu and Zhai [24], this paper aims at providing new results on practical stability of the switched system (1) without a common equilibria and under a time-dependent switching signal. From this general result, we focus on the development of sufficient conditions for practical stability for the important class of switched systems without common equilibria referred in the literature as *switched affine systems* [3,25,7,26,22]. The obtained results for this class of systems are formulated as linear matrix inequalities (LMIs) constraints [4].

This paper is organized as follows. Section 2 provides the definitions of practical stability and the problem formulation. In Section 3, we present the main result on practical stability for the switched system (1) without a common equilibria and considering a time-dependent switching control. Using the result of Section 3, the Section 4 presents sufficient conditions in the form of matrix inequalities constraints for practical stability of switched affine systems. Section 5 presents a illustrative example in the power system stability area and its corresponding results, which demonstrate the effectiveness of the approach proposed in this paper, and Section 6 contains the conclusions and some final remarks on the proposed approach.

2. Preliminaries and problem formulation

In this section we first establish the notation used throughout the paper. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. For two elements a and b in $\mathbb{R}^n$, $\{a,b\}$ denotes the set constituted by only these two elements, while $[a,b]$ denotes the set containing all the points in the line segment between a and b. In addition, $[a,b)$ denotes the set containing all the points of $[a,b]$, except the point b. For matrices and vectors $()'$ means transposition. For a symmetric matrix P, $P \succ 0$ ($P \prec 0$) denotes positive (negative) definiteness. Positive (negative) semi-definiteness is denoted by $P \succcurlyeq 0$ ($P \preccurlyeq 0$). For a set $\Omega \in \mathbb{R}^n$, we use $\overline{\Omega}$ and Ω^c to denote the closure and the complement of Ω, respectively. $N[a,b)$ and $\overline{N}[a,b)$ denote, respectively, the number of switching times on the time interval $[a,b)$ and the maximum number of switching times that can occur on $[a,b)$. For a decimal number r, $\mathrm{int}(r)$ means the integer part of r.

Now, consider again the switched system (1). We do not assume the existence of a common equilibria for all subsystems $\{f_1,\dots,f_N\}$.

The switching signal is defined as

$$\sigma(t) = i_k \in S = \{1,\dots,N\}, \quad \forall t \in [t_k, t_{k+1}), \tag{6}$$

where t_k and t_{k+1} are two consecutive switching times that satisfies

$$t_{k+1} - t_k \ge T, \tag{7}$$

for all the switching times $t_1,\ t_2,\dots,t_k,\ t_{k+1},\dots \in I$ and the index $i_k \in S$ is arbitrarily selected at each of these switching times. Also, T is a positive number denoted in the literature as being a dwell-time of the switching signal $\sigma(t)$ [19].

Definition 1. A positive number T is called a *dwell-time* of the switching signal $\sigma(t)$ if the time interval between any two consecutive switchings k and $k+1$ is no smaller than T.

Since we are not assuming a common equilibria for all subsystems and the switching signal $\sigma(t)$ is only time-dependent, it is not possible to guarantee an asymptotic convergence of the system trajectories to a specific equilibrium point. Hence, stability with respect to a set, rather than a particular point, is studied in this paper in terms of practical stability [27].

Definition 2. The switched system (1) is considered practically stable with respect to the sets $\Omega_1 \subset \mathbb{R}^n$ and $\Omega_2 \subset \mathbb{R}^n$ ($\Omega_1 \subset \Omega_2$) in the time interval $I = [t_0, t_f)$, if $x(t_0) \in \Omega_1$ implies $x(t) \in \Omega_2$, for all $t \in I$.

The first problem under consideration in this paper can be stated as follows.

Problem 1. Given the sets Ω_1 and Ω_2 ($\Omega_1 \subset \Omega_2$) and a time interval $I = [t_0, t_f)$, determine a scalar $T_D > 0$ such that the switched system without a common equilibria (1) is practically stable with respect to Ω_1 and Ω_2 in the time interval I, for every switching signal $\sigma(t)$ satisfying (6) and (7) with a dwell-time $T = T_D$.

From the solution of the Problem 1, we derive some results on practical stability for a particular class of switched systems without a common equilibria, *i.e.*, the switched affine systems described as

$$\dot{x}(t) = f_{\sigma(t)}(x(t)) = A_{\sigma(t)}x(t) + b_{\sigma(t)}, \quad x(t_0) = x_0, \tag{8}$$

where the switching signal $\sigma(t)$ is given by (6) and (7). Assuming that all matrices A_i are nonsingular, each subsystem has a single equilibrium point at $x_{e_i} = -A_i^{-1} b_i$, $i = 1,\dots,N$.

Switched affine systems can arise naturally in many applications, for example, in the presence of saturations or relays. Applications in the area of power electronics on the DC-DC converters control design, for example, are presented in Deaecto et al. [9]. In addition, they can be