Flow Measurement and Instrumentation 50 (2016) 169–177

Contents lists available at ScienceDirect

Flow Measurement and Instrumentation

journal homepage: www.elsevier.com/locate/flowmeasinst

Superior cubic channel section and analytical solution of best hydraulic properties

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ARTICLE INFO

Article history:
Received 27 November 2015
Received in revised form
19 April 2016
Accepted 27 June 2016
Available online 29 June 2016

Keywords:
Channel
Cubic section
Cross-section area
Wetted perimeter
Construction cost
Discharge
Hydraulic
Optimization

ABSTRACT

Channel cross section is an important element in water management. Numerous studies regarding parabolic and semi-cubic sections have been published. This paper introduces a new cubic channel section that has superior properties. Section characteristics are formulated and an explicit approximate formula for the wetted perimeter is derived using the Gauss-Legendre integration method. The theoretical solution of the best hydraulic cubic section is derived using the complex function method and the undetermined Lagrange multiplier optimization algorithm. A comparison with four best hydraulic sections (parabolic, semi-cubic, trapezoidal, and catenary) shows that the discharge of the cubic section is the largest, given the same cross-section flow area and/or wetted perimeter. The results also show that the cubic section is the most economic under the best hydraulic condition. Given its simple form and superior properties, the cubic section should be of interest to water resources engineers.

Notation

The following symbols are used in this paper

A	cross-section flow area (m^2);
A^*	area of excavation (m^2);
A_c^*, A_p^*, P_s^*	area of excavation for the cubic, parabolic, and semi-cubic sections (m^2);
a	shape factor;
B	water surface width (m);
B^*	width of the section at the bank level (m);
B_c^*, B_p^*, B_s^*	width of the section at the bank level for the cubic, parabolic, and semi-cubic sections (m);
β	product of a and h^2;
b	bottom width of trapezoidal section (m);
C	total channel construction cost per unit channel length (CNY);
C_a	cost of land acquisition per unit top width (CNY/m);
C_e	cost of excavation per unit cross-section area (CNY/m^2);
C_l	cost of channel lining per unit perimeter (CNY/m);
f	freeboard (m);
η	optimal ratio of channel water surface width to flow depth;
h	flow depth (m);
h_0	normal depth (m);
h_c	critical depth (m);
i	imaginary unit of a complex number;
m	side slope of the trapezoidal section;
n	Manning roughness coefficient;
P	wetted perimeter (m);
P^*	lining length considering the freeboard (m);
P_c^*, P_p^*, P_s^*	lining length considering the freeboard for the cubic, parabolic and semi-cubic section (m);
Q	flow discharge (m^3/s);
S_0	longitudinal bed slope (m/m);
z	side slope (1 V: z H);
ψ_s	ratio of the discharges of the cubic and semi-cubic sections;
ψ_t	ratio of the discharges of the cubic and parabolic sections

1. Introduction

Channel cross sections are very important in water resources. Favorable section cannot only increase the discharge of transport, but also decrease the costs of construction or reduce the loss of seepage [1]. Clearly, more cross section types provide designers with more choices to adapt channels to different hydraulic, economic, discharge, geotechnical, and hydrogeological conditions.

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http://dx.doi.org/10.1016/j.flowmeasinst.2016.06.019
0955-5986/

The development of new technologies and equipment has made the construction of curved sections easier and more practical. Hence, curved sections have attracted the attention of designers and have been implemented in numerous international projects, including the Pehur High Level Canal in Pakistan and the Genil-Cabra Canal in Spain [2]. The curved sections have the following advantages [3–5]: (a) there are fewer sharp angles and fewer stress concentration points, which ensure that the curve section has fewer cracks and less seepage loss, (b) due to the role of nature, unlined channels tend to approximate a curved shape, (c) curved channels are physically more stable because the side slopes of the channel gradually increases from the bottom to the top, where the slope is maximum, and (d) generally, curved channel sections have better hydraulic characteristics.

Numerous articles about curved sections have been published in the literature and can be grouped into the following types: parabolic, semi-cubic, power law, and composite. For parabolic sections, Loganathan [6] studied the optimal hydraulic parabolic section considering freeboard and water velocity, and obtained the side-slope parameter for the best hydraulic section. Chahar [7] presented the optimal parabolic section for the minimum excavation cost and minimum lining cost. Mironenko [3] presented formulas for the cross-section flow area (or simply flow area), wetted perimeter, and discharge of a parabolic section, along with a comparison with the trapezoidal section. Das [4] presented a section that had parabolic sides and horizontal bottom. His research show this type of section is more economical than the classic parabolic section. Easa [5] went a step further and presented a section with two-segment parabolic sides and horizontal bottom. The results indicate that the section is more economical than a section with one-segment parabolic side.

For semi-cubic sections, Han [8] presented a channel section with semi-cubic sides and horizontal bottom, and deduced section parameters and a formula for the best hydraulic section. Zhao et al. [9] presented an explicit iterative formula for the normal depth of semi-cubic parabolic channels. Power-law channels have been studied by Strelkoff and Clemmens [10] who presented approximate methods for the wetted perimeter. Anwar et al. [2] presented a nonlinear interpolation expression of the wetted perimeter of power-law sections. For composite sections, which are less common in practice, Abdulrahman [11] presented a composite section that consists of a trapezoidal section at the bottom and a rectangular section at the top. The results show that the best hydraulic composite section is more efficient than that of a trapezoidal section. Babaeyan-Koopaei et al. [12] presented a section called parabolic-bottomed triangle cross section that consists of a parabola that opens upward and straight side slopes that are is tangential to the parabola. The authors derived the best hydraulic section demonstrated that the section has better hydraulic characteristics than parabolic and trapezoidal sections.

The current paper presents a new channel section that is based on a cubic function and analytically develops its best hydraulic section. The cubic section is an extension of the previous work on semi-cubic and parabolic sections. A comparison of the proposed cubic section with several existing curved sections in terms of discharge and construction cost is presented. The superiority of the new section in terms of hydraulic and economic performance is demonstrated.

The following sections present the proposed cubic section (including section characteristics, wetted perimeter, and the best hydraulic section) and implementation aspects of the cubic section (computing discharge given flow depth, section design given discharge, and normal and critical depths). Discharge and construction cost comparisons of various section types and application examples are then presented, followed by the conclusions.

2. Proposed cubic channel section

2.1. Section characteristics

The cubic channel section is defined as (Fig. 1a)

$$y = a|x^3| \tag{1}$$

where a = shape factor, x = horizontal coordinate, and y = vertical coordinate. In comparison with the parabolic section and the semi-cubic section (Fig. 1b and c), it is clear that the cubic section has a more flat curved bottom than the parabolic and semi-cubic

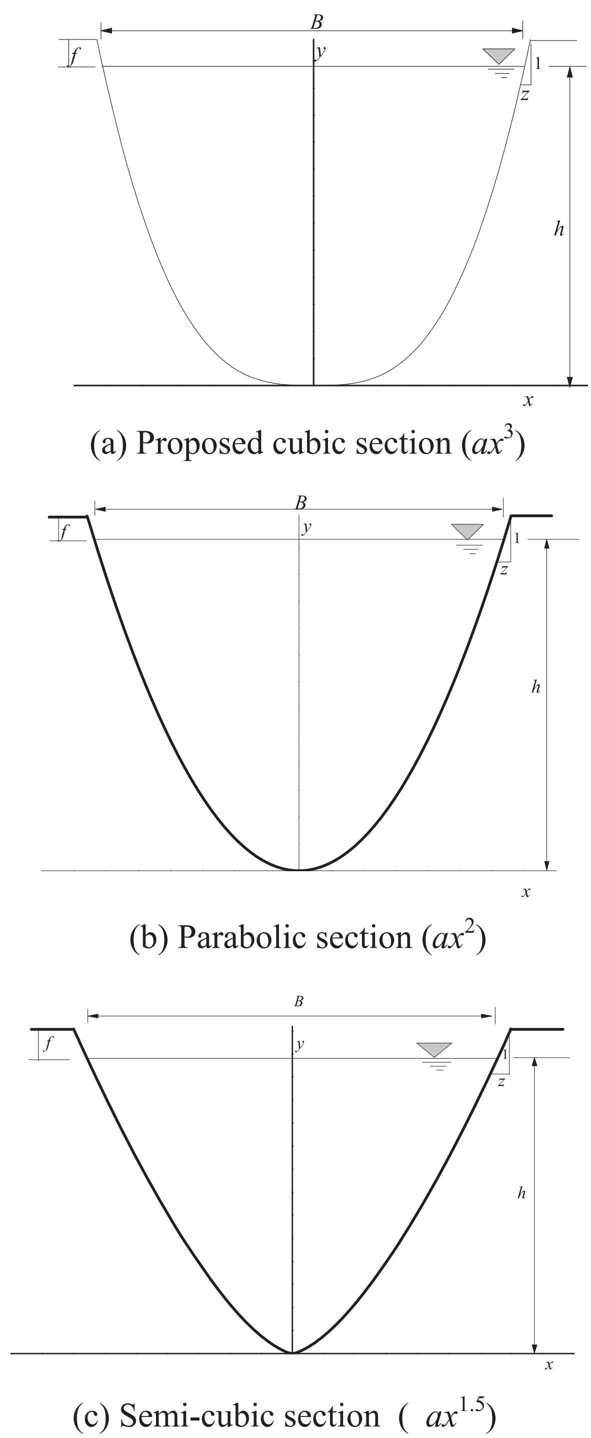

Fig. 1. Comparison of proposed cubic section and existing parabolic and semi-cubic sections.