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Interval observer design for estimation and control of time-delay descriptor systems ☆

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ABSTRACT

The problem of interval observer design is addressed for a class of descriptor linear systems with time delays. First, an interval observation for any input in the system is provided. Second, the control input is designed together with the observer gains in order to guarantee interval estimation and stabilization simultaneously. Efficiency of the proposed approach is illustrated by numerical experiments with the Leontief delayed model.

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1. Introduction

The state estimation problem for uncertain models of industrial plants or biological systems has a great practical importance [2,25,12]. The model uncertainty can be presented by unknown (possibly time-varying) parameters, external disturbances and/or measurement noises. In such a way the designed estimator has to ensure a certain robustness of generated estimates with respect to perturbations. Another issue is that the observer design is structurally complicated in this case, since all uncertain terms should be either estimated simultaneously or avoided in the observer equations (e.g. substituted by some known bounds).

In such a case an important characteristic appears dealing with accuracy of the generated estimates in the presence of all perturbations (unknown parameters, exogenous disturbances, and measurement noise). The problem of accuracy evaluation is partially related with the problem of quantitative estimation of robustness. The difference is that usually for robustness quantification a gain should be computed between the maximal amplitude of perturbation and the maximal amplitude of response (the state estimation error in our case), while for accuracy evaluation the deviations from the nominal values have to be computed as tight as possible. It is strongly appreciated in applications to estimate this accuracy either off-line (during the design phase) or on-line using some numerical routines. The set-membership estimation algorithms dispose this important advantage [15,17,18].

There exist many approaches to design state observers for uncertain systems [2,25,12], all of them are heavily related with the type of the plant model. A special class of models is composed by the so-called (linear) continuous-time descriptor systems (singular systems or differential-algebraic systems) [7]. Descriptor systems have attracted much attention due to the numerous applications in economics (the Leontief dynamic model) [31], in electrical engineering [4], mechanical systems with constraints [30] or flow optimal control [20]. Another important class of models is described by time-delay differential equations [19]. The problem of observer design for delayed systems is rather complex [32], as well as the stability conditions for analysis of functional differential equations are rather complicated [29,13]. Especially the observer synthesis is problematical if the model of a delayed system contains parametric and signal uncertainties, or when the delay is time-varying or uncertain [3,6,38,22,9].

The present work deals with an intersection of these classes, i.e. with linear descriptor systems subject to a constant time delay. An observer solution for this more complex situation may be demanded in many real-world applications (economics, electrical circuits, flow control systems, and so on). Inclusion in a descriptor model of (almost always presented) delay effects increases accuracy of modeling (among others, in the case of convection effects in a fluid flow).

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In addition, a delayed descriptor system is a combination of two dynamics: a differential equation with a difference equation in the delay-free case, algebraic constraint is static), which enlarges the class of physical phenomenon that can be modeled in this framework.

The proposed solution of estimation problem for these systems is based on the set-membership estimation approaches [15,17,18], and in particular a design of interval observers [10,8,15,26,28,24,23] is presented in this work. Some results have been obtained recently on the synthesis of interval observers for time-delay systems [22,9], and the objective is to extend these design tools to the class of descriptor systems. The advantages of the interval observers are that they are well adapted for observer design for highly uncertain systems (if the intervals of admissible values for unknown terms are given) and that they are capable to provide asymptotically rather tight bounds on the estimation accuracy, since the interval of admissible values for the state at each instant of time is evaluated.

The outline of this work is as follows. Some preliminary results are given in Section 2. Problem statement is presented in Section 3. Main results are formulated in Section 4. Numerical experiments and concluding remarks are presented in Sections 5 and 6 respectively.

2. Preliminaries

In the rest of the paper, the following definitions will be used:

- $\mathbb{R}$ is the set of all real numbers ($\mathbb{R}_+ = \{\tau \in \mathbb{R} : \tau \geq 0\}$), $\mathbb{C}$ is the set of complex numbers; $\mathcal{C}_\tau = C([-\tau, 0], \mathbb{R})$ is the set of continuous maps from $[-\tau, 0]$ into $\mathbb{R}$; $\mathcal{C}_{\tau +} = \{y \in \mathcal{C}_\tau : y(s) \in \mathbb{R}_+, s \in [-\tau, 0]\}$.
- x_t is an element of $\mathcal{C}_\tau^n$ associated with a map $x_t : \mathbb{R} \to \mathbb{R}^n$ by $x_t(s) = x(t+s)$, for all $s \in [-\tau, 0]$.
- $|x|$ denotes the absolute value of $x \in \mathbb{R}$, $\|x\|$ is the Euclidean norm of a vector $x \in \mathbb{R}^n$, $\|\varphi\| = \sup_{t \in [-\tau,0]} \|\varphi(t)\|$ for $\varphi \in \mathcal{C}_\tau^n$, $\|A\|$ corresponds to the Euclidean induced norm for a matrix $A \in \mathbb{R}^{n \times n}$.
- For a measurable and locally essentially bounded input $u : \mathbb{R}_+ \to \mathbb{R}^p$ the symbol $\|u\|_{[t_0,t_1]}$ denotes its L_∞ norm $\|u\|_{[t_0,t_1]} = \text{ess}\sup\{\|u(t)\|, t \in [t_0, t_1]\}$, the set of all such inputs u with the property $\|u\|_{[0,+\infty)} < \infty$ will be denoted as $\mathcal{L}_\infty^p$.
- For a matrix $A \in \mathbb{R}^{n \times n}$ the vector of its eigenvalues is denoted as $\lambda(A)$.
- $1_n \in \mathbb{R}^n$ is stated for a vector with unit elements, I_n denotes the identity matrix of dimension $n \times n$.
- $a\mathcal{R}b$ corresponds to an elementwise relation $\mathcal{R} \in \{<, >, \leq, \geq\}$ (a and b are vectors or matrices): for example $a < b$ (vectors) means $\forall i : a_i < b_i$; for $\phi, \varphi \in \mathcal{C}_\tau^n$ the relation $\phi\mathcal{R}, \varphi$ has to be understood elementwise for all domain of definition of the functions, i.e. $\phi(s)\mathcal{R}\varphi(s)$ for all $s \in [-\tau, 0]$.
- The relation $P \prec 0$ ($P \succ 0$) means that the matrix $P \in \mathbb{R}^{n \times n}$ is negative (positive) definite.

2.1. Descriptor linear systems

Consider a descriptor system

$$E\dot{x}(t) = A_0 x(t) + Bu(t),$$

where $x \in \mathbb{R}^n$ and $u(t) \in \mathcal{L}_\infty^m$ are the state and the input (the matrices $E \in \mathbb{R}^{n \times n}$, $A_0 \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$), which is called regular if the characteristic polynomial $\det(sE - A_0)$ does not vanish identically for all $s \in \mathbb{C}$ [14,35]. In the regular case there exist matrices $P, Q \in \mathbb{R}^{n \times n}$ such that

$$QEP = \begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}, \quad QA_0P = \begin{bmatrix} J & 0 \\ 0 & I_{n_2} \end{bmatrix},$$

where $n_2 = n - n_1$ for some $1 \leq n_1 < n$, $N \in \mathbb{R}^{n_2 \times n_2}$ and $J \in \mathbb{R}^{n_1 \times n_1}$ are in the Jordan canonical form, the matrix N is nilpotent of index ν (i.e. $N^\nu = 0$ and $N^{\nu-1} \neq 0$). The descriptor system (or the pair (E, A_0)) has index that is the index of nilpotence ν of N. If a descriptor system has index $\nu > 1$, then it admits impulsive solutions.

2.2. Descriptor linear time-delay systems

Consider a descriptor time-delay system

$$E\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + Bu(t), \tag{1}$$

where $\tau > 0$ is the delay and $x(t) \in \mathbb{R}^n$ is the state, $x_0 \in \mathcal{C}_\tau^n$ is the initial condition, $u(t) \in \mathcal{L}_\infty^m$. The system (1) has an index that is equal to the index of the pair (E, A_0). If $\nu > 1$ then the descriptor time-delay system has impulsive solutions. If a regular descriptor system has index 1, then it can be presented in the following canonical form (similar case has been considered in [14]), where

$$E = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}; \quad A_i = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}, \quad i = 0, 1. \tag{2}$$

Denote in this case $x = [x_1^{\mathrm{T}}\ x_2^{\mathrm{T}}]^{\mathrm{T}}$, where $x_1 \in \mathbb{R}^{n_1}$ and $x_2 \in \mathbb{R}^{n_2}$. Note that this canonical representation is not unique, indeed for any nonsingular matrix $\Upsilon \in \mathbb{R}^{n_2 \times n_2}$,

$$E = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} B_1 \\ \Upsilon B_2 \end{bmatrix}; \quad A_i = \begin{bmatrix} A_{i1} & A_{i2} \\ \Upsilon A_{i3} & \Upsilon A_{i4} \end{bmatrix}, \quad i = 0, 1$$

is again a canonical representation of (1) in the same coordinates x. If it is assumed that $\det(A_{04}) \neq 0$ (that is the case considered below), then (3) can be selected in the following form for $\Upsilon = -A_{04}^{-1}$:

$$E = \begin{bmatrix} I_{n_1} & 0 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}; \quad A_0 = \begin{bmatrix} A_{01} & A_{02} \\ A_{03} & -I_{n_2} \end{bmatrix}, \quad A_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{bmatrix}. \tag{3}$$

Proposition 1 (*Fridman and Shaked [14]*). *Assume that* $\det(A_{04}) \neq 0$ *and* $x_0 \in \mathcal{C}_\tau^n$, *then for any* $u(t) \in \mathcal{L}_\infty^m$ *an absolute continuous solution (see [4]) of (1) exists for all* $t \in \mathbb{R}^+$ *and it is unique.*

The notion of (global) asymptotic stability for (1) is understood in the standard for time-delay systems sense [19].

Proposition 2 (*Fridman and Shaked [14]*). *Assume that* $\det(A_{04}) \neq 0$ *and* $\max_{1 \leq i \leq n_2} |\lambda_i(A_{04}^{-1} A_{14})| < 1$. *If there exists* $P \in \mathbb{R}^{n \times n}$,

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1 = P_1^{\mathrm{T}} \succ 0, \quad P_1 \in \mathbb{R}^{n_1 \times n_1}, \quad P_3 \in \mathbb{R}^{n_2 \times n_2}, \tag{4}$$

and $U \in \mathbb{R}^{n \times n}$, $U = U^{\mathrm{T}} \succ 0$ *that satisfy the following LMI for some* $\gamma > 0$:

$$\begin{bmatrix} \Psi & P^{\mathrm{T}}B & P^{\mathrm{T}}A_1 \\ B^{\mathrm{T}}P & -\gamma^2 I_m & 0 \\ A_1^{\mathrm{T}}P & 0 & -U \end{bmatrix} \prec 0,$$

$$\Psi = P^{\mathrm{T}}A_0 + A_0^{\mathrm{T}}P + U + I_n,$$

and any delay $\tau > 0$, *in addition its* H_∞ *gain from the input* u *to the state* x *is less than* γ.

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