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Temporal stabilizability and compensatability of time-varying linear discrete-time systems with white stochastic parameters

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ABSTRACT

This paper reveals that apart from changes of system structure vital system properties such as stabilizability and compensatability may be lost *temporarily* due to the stochastic nature of system parameters. To that end *new system properties* called temporal mean-square stabilizability (tms-stabilizability) and temporal mean-square compensatability (tms-compensatability) for time-varying linear discrete-time systems with white stochastic parameters (multiplicative white noise) are developed. When controlling such systems by means of (optimal) state feedback, tms-stabilizability identifies intervals where *mean-square stability* (*ms-stability*) *is lost temporarily*. This is vital knowledge to both control engineers and system scientists. Similarly, tms-compensatability identifies intervals where ms-stability is lost temporarily in case of full-order (optimal) output feedback. Tests explicit in the system matrices are provided to determine each temporal system property. These tests compute *measures* of the associated temporal system properties. Relations among the new system properties as well as relations with associated existing system properties are investigated and established. Examples illustrating principal applications and practical importance are provided.

1. Introduction

Research performed during the last decade showed that the *structure* of time-varying linear systems may change or almost change [28–32]. These changes of structure cause and explain differences between reachability and controllability and dually observability and reconstructability. They lead naturally to the definition of *temporal linear system structure* and associated *temporal properties* like temporal controllability and reconstructability. These temporal properties reveal *time intervals* where the associated ordinary system properties are *lost temporarily*. Obviously this is vital knowledge to control engineers and system scientists.

In continuous-time the intervals and associated changes of structure are detected by the differential Kalman decomposition [28,29]. In discrete-time they are detected by the j-step, k-step Kalman decomposition [30]. If controllability is lost temporarily over an interval, it is important to check whether or not stabilizability is lost temporarily over that interval. Developing and verifying temporal stabilizability was done in [31,32]. Dually temporal reconstructability and detectability were also developed and verified in these papers. These important, practical developments have sometimes been criticized because of their dependence on the selected state-vector norm. Recently it was explained in [33] that this dependence is inevitable when analysis is restricted to finite time-intervals. Also sensible choices of the state-vector norm were presented and discussed in [33].

This paper reveals that apart from changes of system structure, the *white stochastic nature of system parameters* can cause temporal loss of vital system properties, notably stabilizability and compensatability i.e. the ability to stabilize a system by means of state and output feedback respectively. To that end this paper *extends* temporal properties that have been introduced for time-varying linear discrete-time systems with deterministic parameters to systems having white stochastic parameters. Also a *new temporal system property* is introduced called *temporal compensatability*. Ordinary compensatability was introduced because separability and duality between estimation and control are lost if system parameters become stochastic [9]. If system parameters are deterministic, compensatability is equivalent with stabilizability plus detectability, otherwise it is stronger. This paper reveals that, even if the parameters are deterministic, temporal compensatability is still stronger than temporal stabilizability plus temporal detectability. Roughly speaking this is because the time needed for LQG compensators to start convergence is about the *sum* of the times needed by the state estimator and controller to start convergence.

Discrete-time linear systems with white stochastic parameters offer a way to design *non-conservative robust digital feedback controllers* [6,7,38,39]. These controllers may be *perturbation feedback controllers* based on *linearized dynamics* about possibly optimal state trajectories associated with a non-linear system. The perturbation

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feedback controllers as well as the possibly optimal state trajectories may be computed *off-line*. This enables the handling of a wide range of constraints and optimization criteria offering a wide range of application [5]. The linearized dynamics used for perturbation feedback controller design are generally *time-varying*. This is one important reason to study time-varying linear discrete-time systems with white stochastic parameters (multiplicative white noise). Other reasons are that discrete-time system parameters may be white due to stochastic sampling, randomly varying delays or Markovian jumps of system structure [4,11,17–19,22,24–26]. Optimal state and output feedback control of linear systems with white stochastic parameters has been addressed in [8,10,14–16,23,27].

The authors are aware of one other development that considers stability and stabilizability, but not compensatability, over finite time intervals. Two properties called finite-time stability and stabilizability have been introduced and investigated [1–3]. Like our temporal system properties they apply to finite time-intervals. Finite-time stability and stabilizability consider a state-vector norm over the full finite time-interval whereas temporal stability and stabilizability consider a state-vector norm at the initial and final time of the interval only. By shifting the initial time of the interval towards the final time however, a similar picture of closed loop stability over the full finite time interval is obtained [31,32]. On the other hand temporal stability and stabilizability consider arbitrary initial conditions whereas finite-time stability and stabilizability are defined for fixed initial conditions only. Moreover finite-time stability and stabilizability computations concern LMI's instead of standard LQ computations required by temporal stability and stabilizability when system parameters are deterministic.

To analyze the effect of stochastic parameters on stability, stabilizability and compensatability of systems the mean-square (ms) of the state must be considered. Temporal mean-square stabilizability (tms-stabilizability) identifies temporal loss of closed loop mean-square stability in case of (optimal) full state feedback. It is presented in Section 3. Temporal mean-square compensatability (tms-compensatability) does the same in case of (optimal) full-order output feedback and is presented in Section 4. In both sections important relations among these new system properties are established. Also relations with existing system properties, partly relating to linear systems with deterministic parameters, are established. Examples illustrating these relations are presented in Section 5. First however Section 2 presents a semi-industrial example to illustrate the main contribution and practical importance of the results developed in this paper. Conclusions are drawn in Section 6 an important one being that tms-stabilizability and tms-compensatability are most important for feedback control design based on time-varying linear dynamics with stochastic parameters.

2. Illustrative example

The new results and temporal system properties will be presented in the next two sections. In this section the main contribution of this paper and one of its major applications is illustrated and demonstrated first. This is done by means of a semi-industrial example.

Example 1. Consider the digital optimal perturbation feedback control of the "Goddard Rocket" around its optimal trajectory as presented in [5], Example 2. The example considered here is identical except for the parameters of the equivalent discrete time-varying linearized system (EDTVLS) used for digital optimal perturbation feedback design. These are turned into *stochastic parameters* using a possibly time-varying *parameter uncertainty measure* $\beta_i \geq 0$, where i denotes discrete-time. When $\beta_i = \beta = 0$ the parameters are deterministic at each time i and the results of Example 2 presented in [32] are obtained. With increasing β_i, parameter uncertainty at time i increases.

Fig. 1 presents values of the temporal mean-square stabilizability measure $\rho_{min}^{tms}(i,25)$, $i = 0, 1, .., 24$ of the closed loop system with full state feedback over time-interval $(i,25)$. If the value falls below one, the system is temporal mean-square stabilizable (tms-stabilizable) over time-interval $(i,25)$. For clarity the results are plotted using both a linear and logarithmic scale. Similarly Fig. 2 presents values of the temporal mean-square compensatability measure $\sigma_{min}^{tms}(i,25)$, $i = 0, 1, .., 24$ of the closed loop system with full-order output feedback over time-interval $(i,25)$. Again if the value falls below one, the system is temporal mean-square compensatable (tms-compensatable) over time-interval $(i,25)$. As expected, with increasing constant values of β, i.e. with increasing parameter uncertainty at each time i, tms-stabilizability and tms-compensatability become worse because their measures increase. Also observe that tms-stabilizability is far better than tms-compensatability. This represents the well-known fact that full state feedback is to be preferred over full-order output feedback. A time-varying uncertainty measure β_i may be used to indicate or describe time-varying levels of model parameter uncertainty. Observe from Figs. 1 and 2 that introducing stochastic parameters, e.g. to promote robustness, goes at the expense of tms-stabilizability and tms-compensatability.

3. Temporal stabilizability

The possibility of time-varying linear systems with deterministic parameters to change or almost change structure motivated the investigation into temporal properties of these systems [28–32]. A change of structure comes generally with a change of important

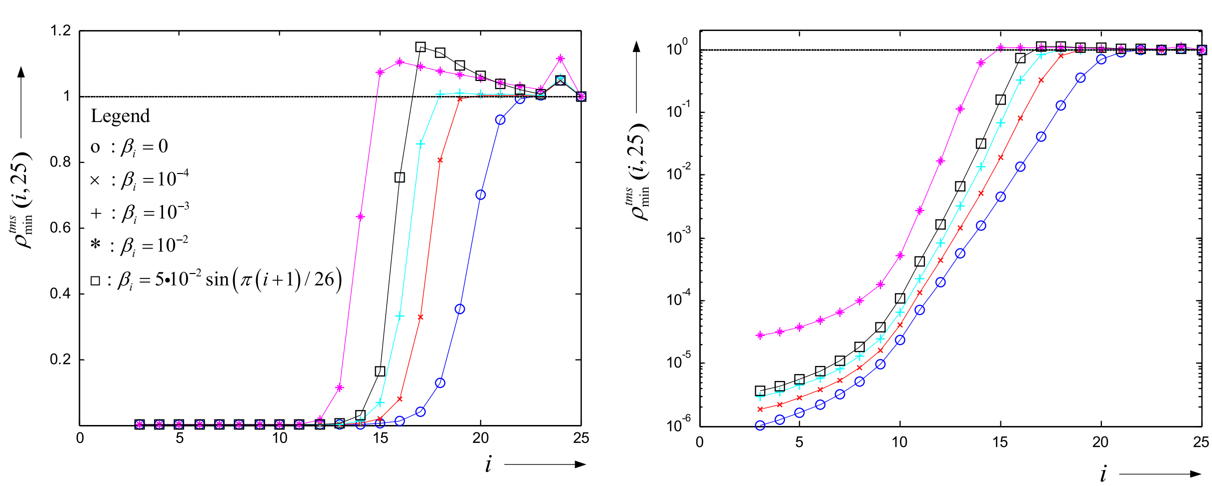

Fig. 1. tms-Stabilizability measures Example 1 for different values of β_i.