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On active disturbance rejection control for nonlinear systems using time-varying gain ☆

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ABSTRACT

In this paper, we propose an modified nonlinear extended state observer (ESO) with a time-varying gain in active disturbance rejection control (ADRC) to deal with a class of nonlinear systems which are essentially normal forms of general affine nonlinear systems. The total disturbance which includes unknown dynamics of the system, external disturbance, and unknown part of the control coefficient is estimated through ESO and is canceled in nonlinear feedback loop. The practical stability for the resulting closed-loop is obtained. It is shown that the "peaking value" occurred often in the constant high gain design can be significantly reduced by the time-varying gain approach.

1. Introduction

In the past three decades, many control approaches have been developed to cope with system uncertainty or external disturbances. Basically, there are two types of strategies to deal with uncertainty. The first one focuses on the worst case scenario which makes the controller designed conservative. This can be found in the sliding mode control and the high gain control, among many others. The other is first to estimate uncertainty and then cancel the effect of uncertainty in feedback loop. The latter idea can be found typically in adaptive control, the internal model principle, and external principle [21]. However, in adaptive control, the updated parameters are not always convergent, and the internal and external model principle requires priority knowledge of dynamics of unknown disturbance.

The active disturbance rejection control (ADRC), as an unconventional design strategy similar to the external model principle [21], was first proposed by Han in 1998 based on realistic rethinking about the PID technology that has dominated the control engineering for almost one century [13]. The uncertainties dealt with by ADRC are much more complicated. For instance, ADRC can deal with the coupling between the external disturbances, the system un-modeled dynamics, and the superadded unknown part of control input. The most remarkable feature of ADRC is that the disturbance is estimated, in real time, through an extended state observer and is canceled in the feedback loop. This reduces significantly the control energy in practice [32].

In the past two decades, ADRC has been successfully applied to many engineering control problems as reviewed in hysteresis compensation [7], high pointing accuracy and rotation speed [20], noncircular machining [26], fault diagnosis [27], high-performance motion control [23], chemical processes [28], vibrational MEMS gyroscopes [29,31], tension and velocity regulations in Web processing lines [14], DC–DC power converter [24], among many others. In all applications in process control and motion control, compared with the huge literature of control theory in dealing with uncertainty such as system un-modeled dynamics [5], external disturbance rejection [4], and unknown parameters [3], the ADRC has exhibited remarkable characteristics of independent of mathematical modes like PID-control; whether it is on high accuracy control of micron grade or integrated control of very large scale. It is now generally acknowledged that the ADRC is a new control strategy that is capable of dealing with un-modeled dynamics and external disturbance, regardless of nonlinearity, time-variance in systems. For instance, by the internal model principle for output regulation, it requires the availability of disturbance dynamics whereas by ADRC, only upper bound of external disturbance is needed.

The design of ADRC can be split into three steps. The first step is to design a tracking differentiator (TD). Since in many practical applications, we know only reference signal v itself, while in feedback, we

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need the derivatives of v. This gives rise to differential tracking problem. Han initially proposed a noise-tolerance TD to recover the derivatives of the reference signal v [11]. For general treatment on differential tracking, we refer to [19] and the references therein. The second step towards ADRC is to design an "extended state observer" (ESO) to estimate not only the state of system but also the total disturbance. The last step is the ESO-based feedback control [10], which is somehow the separation principle in nonlinear system control.

The convergence of linear/or nonlinear TD is reported in [8,10,12]. The convergence of linear ESO is available in [30], and very recently, nonlinear ESO has been studied in [10] where linear ESO is a special case of the nonlinear ones. It is shown that with an appropriate choice of nonlinear functions in ESO such as weighted homogeneous functions can improve the accuracy and reduce the peaking value, with the same constant high gain [10]. The convergence of linear ADRC, which is based on linear ESO and linear feedback, is investigated in [15] and the convergence of nonlinear ADRC has been proven in [9]. In all these works, no matter linear or nonlinear, the ESO uses constant high gain tuning parameter. However, the constant high gain tuning parameter causes notorious "peaking value problem" in the initial time stage. In [6], a saturated function method is applied to reduce the peaking value but the bound of initial values is assumed preliminarily.

In this paper, we consider the following nonlinear system:

$$\begin{cases} \dot{x}(t) = A_n x(t) + B_n[f(t,x(t),\zeta(t),w(t)) + b(t)u(t)], \\ \dot{\zeta}(t) = f_0(t,x(t),\zeta(t),w(t)), \\ y(t) = C_n x(t), \end{cases} \tag{1.1}$$

where $x \in \mathbb{R}^n$ and $\zeta \in \mathbb{R}^m$ are the system states, $w \in \mathbb{C}^1([0,\infty),\mathbb{R})$ is the external disturbance, A_n, and B_n are defined as

$$A_n = \begin{pmatrix} 0 & I_{n-1} \\ 0 & 0 \end{pmatrix}, \quad B_n^\top = C_n = (0,0,\ldots,1), \tag{1.2}$$

$f \in \mathbb{C}(\mathbb{R}^{n+m+2},\mathbb{R})$, and $f_0 \in \mathbb{C}(\mathbb{R}^{n+m+2},\mathbb{R}^m)$ are unknown nonlinear functions, $u(t) \in \mathbb{R}$ is the input (control), and $y(t) = C_n x(t) = x_1(t)$ is the output (measurement). The control coefficient $b(t)$, with nominal value $b_0 \neq 0$, contains some uncertainty.

System (1.1) is quite general. It is actually the normal form of $(n+m)$-order affine nonlinear systems with relative degree n. According to [16], if a general $(n+m)$-order affine nonlinear system

$$\begin{cases} \dot{\chi} = \phi(\chi) + \varphi(\chi)u, \\ y = h(\chi) \end{cases}$$

has a relative degree n, then it can be transformed into the form

$$\begin{cases} \dot{x} = A_n x + B_n[f(x,\zeta) + b(x,\zeta)u], \\ \dot{\zeta} = f_0(x,\zeta), \\ y = C_n x. \end{cases}$$

System (1.1) may also arise in models of mechanical and electromechanical systems. Examples can be found in [20,24,26,32,31,28].

We design a nonlinear ESO with time-varying gain for system (1.1) as follows:

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + \dfrac{1}{r^{n-1}(t)} g_1(r^n(t)(\delta(t)), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) + \dfrac{1}{r^{n-2}(t)} g_2(r^n(t)\delta(t)), \\ \vdots \\ \dot{\hat{x}}_n(t) = \hat{x}_n(t) + 1(t) + g_n(r^n(t)\delta(t)) + b_0 u(t), \\ \dot{\hat{x}}_{n+1}(t) = r(t) g_{n+1}(r^n(t)\delta(t)), \end{cases} \tag{1.3}$$

where $\delta(t) = \hat{x}_1(t) - y(t)$, and $r(t)$ is the time-varying gain to be increased gradually. When $r(t) \equiv 1/\varepsilon$, (1.3) is reduced to the constant high gain nonlinear ESO in [10]. The aim of ESO is to estimate the states $x_1, x_2, \ldots x_n$ and total disturbance

$$x_{n+1}(t) \triangleq f(t,x(t),\zeta(t),w(t)) + [b(t) - b_0]u(t), \tag{1.4}$$

which is also called the extended state.

The high gain observer has been studied extensively. A recent work is reviewed in [18]. Observer with time-varying gain (updated gain or dynamic gain) is also used in [1,2,22], where the gain is a dynamics determined by some nonlinear function related to control plant. The choice of our time-varying gain is flexible. Basic requirement is that the time-varying gain should grow from a small value to maximal value to reduce the peaking value. The major difference between [1,2] and this paper is that there is no estimation for uncertainty in these works. Only in [22], a *constant* unknown nominal control value is estimated on stabilization for an affine nonlinear system. The estimation/cancelation nature of ADRC makes it very different.

The main contribution of this paper is that we introduce a type of time varying-gain in observer (1.3) to achieve peaking value reduction observed by the constant high gain in [10]. In addition, the nonlinear functions in (1.1) can be Hölder continuous rather than Lipschitz continuous assumed in our previous works [10,9].

We proceed as follows. In Section 2, we give the main results of ESO (1.3) based feedback control. Some numerical simulations are presented for illustrations. The proof for the main results is presented in Section 3.

2. Main results

Let us first recall the whole process of ADRC for system (1.1). The first part of ADRC is tracking differentiator (TD). For a reference signal v, we use the following TD to estimate its derivatives [11]:

$$\begin{cases} \dot{z}_1(t) = z_2(t), \\ \vdots \\ \dot{z}_n(t) = z_{n+1}(t), \\ \dot{z}_{n+1}(t) = -\rho^{n+1} k_1(z_1(t) - v(t)) - \rho^n k_2 z_2(t) - \cdots - \rho k_{n+1} z_{n+1}(t). \end{cases} \tag{2.5}$$

By Theorem 3.1 of [11], if $\sup_{t \in [0,\infty)} |v^{(i)}(t)| < \infty$, $i = 1,2,\ldots,n$, and the following matrix is Hurwitz:

$$\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_n \end{pmatrix}$$

then for any $a > 0$, $|z_i(t) - v^{(i-1)}(t)| \le \overline{M}/\rho$, $i = 1,2,\ldots,n+1$ uniformly in $t \in [a,\infty)$, where $\overline{M}$ is a ρ-independent constant. It is noted that if the derivatives of v are available, we just let $z_i = v^{(i-1)}$.

Since the TD part is relatively independent of other two parts of ADRC, we do not couple TD in the closed loop; instead, we use z_i directly in the feedback loop.

The second part of ADRC is the ESO (1.3) that estimates both state and the total disturbance of system (1.1).

Suppose that we have obtained estimates for both state and total disturbance. We then use estimation/cancelation strategy to design the ESO-based output feedback control as follows:

$$u(t) = \frac{1}{b_0}(u_0(\hat{x}_1(t) - z_1(t), \ldots, \hat{x}_n(t) - z_n(t)) + z_{n+1}(t) - \hat{x}_{n+1}(t)), \tag{2.6}$$

where $\hat{x}_{n+1}$ is used to compensate the total disturbance x_{n+1} and u_0 is the nominal control to be specified later. The objective of the control is to make the error $(x_1(t) - z_1(t), x_2(t) - z_2(t) \ldots, x_n(t) - z_n(t))$ be convergent to zero as time goes to infinity in the prescribed