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Cooperative economic model predictive control for linear systems with convex objectives ☆, ☆☆

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ABSTRACT

In this paper we propose a cooperative distributed economic model predictive control strategy for linear systems which consist of a finite number of coupled subsystems. The suggested feedback strategy is generating control input which converges to a set of Nash equilibria of the corresponding game provided infinite iterations are allowed at each sampling time. Moreover, the control for each subsystem is computed in itself without coordination layer except for a synchronization requirement between subsystems.

We first introduce distributed linear systems with two subsystems and economic model predictive control, then show the convergence and stability properties of a suboptimal model predictive control strategy for the system. The optimization problem for the implementation of MPC is stated with a terminal equality constraint and a terminal cost.

1. Introduction

Model predictive control (MPC) is a feedback design technique which computes control actions by taking into account the current state of a plant and all sorts of constraints between input and output variables that need to be fulfilled. Typically, a cost functional is available or is suitably designed so as to find the most appropriate control action by means of real-time optimization. At the same time the control action should steer the plant's operation to a desired operating condition within reasonable amount of time.

Recently, as an application for systems which consist of multiple subsystems and/or large-scale systems, distributed MPC has been investigated. See [19] for a recent survey on the subject. The present note further develops the so-called *cooperative* Model Predictive Control; this is a particular variant of MPC in which it is assumed that individual subsystems may cooperate towards a common objective. In [20] a solution for tracking MPC of distributed systems as suboptimal MPC was suggested and analyzed.

The contribution of this article is to extend the techniques suggested in [20] to the case of economic MPC [17]. This is a variant of standard MPC which aims at achieving both transient and steady-state costs minimization simultaneously. In particular, the MPC control layer directly uses the true economic cost in devising the optimal control action; this entails that cost need not be minimal at the best steady-state and, as a consequence, overall stability may be affected.

Recently, average performance and stability issues as well as Lyapunov-based analysis techniques were proposed in [3] and [6] respectively. While control algorithms were initially designed by making use of terminal equality constraints (for the sake of guaranteeing both recursive feasibility and performance bounds), subsequent works have relaxed in several directions this assumptions, for instance by using terminal penalty functions [1], generalized terminal constraints [7] and [12] or by removing terminal equality constraints [8] (see [9] for a distributed approach to this problem).

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In this paper we propose a distributed economic MPC problem for linear systems with economic cost function, and also suggest a method for its solution.

2. Linear distributed systems

We assume a discrete-time linear system whose input consists of two components. Although all arguments can be extended to the case of M components with ease (see Section 8.2), we limit ourselves to this case for notational simplicity:

$$x^+ = Ax + Bu = Ax + [B_1 \quad B_2]\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{1}$$

where $x \in \mathbb{X} \subset \mathbb{R}^n$, $u_1 \in \mathbb{U}_1 \subset \mathbb{R}^{m_1}$, $u_2 \in \mathbb{U}_2 \subset \mathbb{R}^{m_2}$, $A \in \mathbb{R}^{n \times n}$, $B_1 \in \mathbb{R}^{n \times m_1}$ and $B_2 \in \mathbb{R}^{n \times m_2}$. We assume that the state and input set $\mathbb{X}$, $\mathbb{U}_1$ and $\mathbb{U}_2$ are all compact. Notice that distributed systems with two subsystems also can be represented as in (1) since coupled states are included in A. Therefore the subsystems are coupled through states as well as input.

Throughout this paper by cooperative MPC we mean that the two controllers share a common cost to minimize. That is, we aim to separately design control laws $u_1(k) = \kappa_1((x(k))$ and $u_2(k) = \kappa_2(x(k))$ which optimize the given cost cooperatively. As a design technique we adopt economic model predictive control which is presented specifically in the next section.

3. Economic model predictive control

For systems as expressed in (1), pointwise state and input constraints are introduced. If the following constraints hold for the state variable $x(\cdot)$ and input $u(\cdot)$ we say that the pair $(x(\cdot), u(\cdot))$ is feasible:

$$\begin{aligned} &x(k+1) = Ax(k) + Bu(k) \\ &g(x(k), u(k)) \le 0, \ \forall k \in \mathbb{I}_{\ge 0} \end{aligned} \tag{2}$$

We assume that $g : \mathbb{X} \times \mathbb{U} \times \mathbb{U}_2 \to \mathbb{R}$ is a convex function, so any sublevel set is also convex.

For the systems and constraints we define the following objective function with a stage cost function $\ell(x, [u_1, u_2]) : \mathbb{R}^n \times \mathbb{R}^{m_1} \times \mathbb{R}^{m_2} \to \mathbb{R}$ which is assumed strictly convex:

$$\sum_k \ell(x(k), [u_1(k), u_2(k)]) \tag{3}$$

Given the convexity of $\ell(\cdot)$ and linearity of (1), we will design a controller which operates the system at the best admissible steady-state at least asymptotically. The best admissible steady-state is defined as the solution of the following steady-state optimization problem:

$$\min_{x,u} \ell(x, u) \text{ subject to } x - (Ax + Bu) = 0, \ g(x, u) \le 0. \tag{4}$$

Note that, unlike standard model predictive control, in economic model predictive control there might exist pairs (x,u) such that $\ell(x, u) \le \ell(x_s, u_s)$ and $g(x, u) \le 0$ if (x,u) is not a itself a steady-state.

Now, to study the issue of feasibility for the optimization problem related to MPC it is useful to define the following sets.

Definition 1 (*Feasible set*). We define feasible set $\mathbb{Z}_N$ for economic MPC as the set of $(x, \mathbf{u})$ pairs detailed below:

$$\begin{aligned} \mathbb{Z}_N := \{&(x, \mathbf{u}) \in \mathbb{R}^n \times \mathbb{R}^{N(m_1+m_2)} | x(0) = x, \\ &x(N) \in \mathbb{X}_f, x(k+1) = Ax(k) + Bu(k), \\ &g(x(k), u(k)) \le 0, \ \forall k \in \mathbb{I}_{0:N-1}\} \end{aligned} \tag{5}$$

where $\mathbb{X}_f \subseteq \mathbb{X}$ is a convex and compact control positively invariant set which contains x_s and $\mathbf{u}$ is a sequence of $u(k)$, $k \in \mathbb{I}_{\ge 0}$ i.e, $\mathbf{u} = \{u(0), u(1), \dots, u(N-1)\}$.

The projection of $\mathbb{Z}_N$ onto $\mathbb{X}$ is denoted as the set of feasible states.

Definition 2 (*Feasible states*). $\mathcal{X}_N$ is called the set of feasible states, and is defined as follows:

$$\mathcal{X}_N := \{x | \exists \mathbf{u} \quad \text{such that} \quad (x, \mathbf{u}) \in \mathbb{Z}_N\}. \tag{6}$$

For each feasible state x we also define set of feasible control sequences.

Definition 3 (*Set of feasible control sequences*). For a given feasible state $x \in \mathcal{X}_N$ the set of feasible control sequences is defined as

$$\mathcal{U}_N(x) := \{\mathbf{u} | (x, \mathbf{u}) \in \mathbb{Z}_N\}$$

We introduce the following well-known convexity property of the feasible set [4].

Lemma 1. *The feasible set $\mathbb{Z}_N$ is convex.*

It is worth pointing out that since there is no termination time in the operation of real plants, we first solve the optimization Problem 3 over finite time horizon, then proceed in a receding horizon manner by shifting forward by one interval at each sampling time the overall optimization problem as customary in MPC. We will provide more details about this later in Section 4.

3.1. Suboptimal MPC

Implementation of distributed MPC strategies through multiple iterations is a process similar to the computation of centralized MPC through distributed optimization over many processors. Since MPC is based on the assumption of on-line or real-time implementation of an optimal solution, the available time for computation is limited. Therefore, despite convexity of the underlying problem could in principle lead to asymptotically convergent solutions, we allow the implementation of feasible suboptimal control actions instead of the true optimal control. This, at least, if the optimization is too complex to solve within the available time.

For the current feasible state $x \in \mathcal{X}_N$ we assume a feasible control sequence $\tilde{\mathbf{u}} \in \mathcal{U}_N(x)$. After iterating several times a certain optimization problem and suitably updating the best available guess of its optimal solution (this is specified in Section 8), each subsystem generates a new input sequence $\mathbf{u}_i$ giving rise to improved performance with respect to the warm start input trajectory which is computed at the previous time-instant. We inject the first component $u(0)$ of $\mathbf{u}$, which is constructed as $[\mathbf{u}_1(0), \mathbf{u}_2(0)]$, to the plant, and the next state arises according to the evolution $x^+ = Ax + Bu$. At the following sampling time we indicate $\tilde{\mathbf{u}} = \{u(1), u(2), \dots, u(N-1), \kappa_f(x(N))\}$ as a warm start to be transmitted to and to be used as an initial guess by each subsystem. The local controller κ_f simply fulfills $\kappa_f(x_s) = u_s$ for the case of MPC subject to terminal equality constraint and is otherwise defined to be a feasible control action making the set $\mathbb{X}_f$ positively invariant.

4. Formulation of centralized and cooperative economic MPC

Let us now introduce the optimization problems associated with centralized and cooperative MPC so that comparison between the two could clarify the role of each subsystem in the plantwide system. As stated in the previous section, there exists a shared objective function among all subsystems, which we define as a sum of stage cost functions $\ell(x(k), u_1(k), u_2(k))$ over a fixed