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Attitude synchronization of satellites with internal actuation

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ABSTRACT

We present control synthesis for attitude synchronization of a cluster of satellites, that are internally actuated, using a coordinate-free approach. The control law is designed using the notion of potential energy shaping that rests on the use of modified trace functions. The distinguishing aspect of the internal actuation model is that the drift vector field in the system dynamics explicitly depends on the *conserved total angular momentum*. The control law presented is applicable to a general undirected graph of communication between the satellites, with one satellite of the cluster communicating with the leader. Further, the control law is applicable with just two vector observations in each satellite, and hence making redundant, complete attitude information.

1. Introduction

Aerospace applications that involve changing the orientation of a satellite or a spacecraft, and applications like underwater vehicles and robots, have motivated the research community to examine the rigid body attitude control problem. A few papers that discuss attitude control are [22,11,14,31,1,8,5,9,18,19]. The implementation of attitude control is achieved either by using external actuators (gas jets) or internal actuators (control moment gyros, reaction wheels).

Attitude manoeuvres are required when the rigid body is required to point in a certain direction to collect or transmit information. The use of a single satellite to collect and transmit information may increase the weight substantially due to the hardware that it has to carry onboard. Further, failure of one satellite could lead to failure of the mission and hence the overall objective. In certain cases there may be limitations on the size of the satellite which in turn limits the number of sensors that can be mounted on it. Motivated by such situations and due to the benefits of a multi-satellite system over a single satellite, cooperative attitude synchronization has gained considerable attention in the recent past.

With the benefits of a multi-satellite system, design of consensus algorithms for such satellite systems has become necessary to achieve a desired objective. In satellite formation *attitude synchronization* is often a desired objective. When all the rigid bodies in the system under cooperation converge to the same orientation (or attitude) asymptotically it is termed attitude synchronization. Attitude synchronization is a form of consensus where the desired state is the attitude of the rigid body. There has been extensive research on various consensus algorithms in this area. A few papers that discuss certain consensus algorithms for point mass agents are [25–27,13,21,30,12,17]. The rigid body attitude synchronization problems under different communication topologies have been studied by a few groups. In [6,16] the authors have discussed a leader–follower approach for attitude synchronization of a rigid body when the angular velocity measurements are not available. The papers [23,24] discuss energy shaping methods for synchronization. A leaderless attitude synchronization algorithm has been designed and discussed in [28,29]. In [20,32], the authors have discussed relative attitude control using line-of-sight measurements. These papers focus on the design of control laws for rigid bodies with external actuation schemes.

Motivated by the ongoing research on attitude synchronization, and design of control algorithms for rigid bodies with internal actuation [5,15,3] we explore the problem of attitude synchronization with multiple satellites where each satellite is actuated internally. The distinguishing aspect of the internal actuation model is that the drift vector field in the system dynamics explicitly depends on the *conserved total angular momentum*. In [15], the authors discuss a control law based on backstepping and observer design for an internally actuated system. In the present work, a coordinate-free approach for design of control law and stability analysis of the system has been presented. A leader–follower strategy is used to design the control algorithm to synchronize the attitude of all satellites to a desired orientation

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exponentially. The satellites come to rest as they converge to the desired attitude. We assume that the first satellite has information about the desired attitude. Satellite (or agent) i has relative attitude information with respect to its nearest neighbor $(i-1)$. The control law is designed using the notion of *potential energy shaping* that rests on the use of *modified trace functions*. The control law presented is applicable to a general undirected graph of communication between the satellites, with one satellite of the cluster communicating with the leader. Further, the control law is applicable with just two vector observations in each satellite, and hence making redundant, complete attitude information.

The paper is organized as follows: Section 2 presents some mathematical preliminaries necessary for the current work. In Section 3 the objective and the vehicle model are discussed. The potential function constructed from modified trace function and the control law derived from it are presented in Section 4. The control law in Section 4 is generalized for a general communication graph in Section 5. Section 6 presents the control law based on vector observations made by the satellites. In Section 7 simulation results are discussed, and Section 8 summarizes the work.

2. Mathematical preliminaries

The trace of a matrix defined as $\mathrm{tr}:\mathbb{R}^{n\times n}\to\mathbb{R}$ is

$$\mathrm{tr}(Q)=\sum_{i=1}^{n} q_{ii},$$

where $Q=[q_{ij}]_{n\times n}$. The map $\mathrm{diag}:\mathbb{R}^3\to\mathbb{R}^{3\times 3}$ such that for every $a\in\mathbb{R}^3$

$$\mathrm{diag}(a)=\begin{bmatrix} a_1 & 0 & 0\\ 0 & a_2 & 0\\ 0 & 0 & a_3\end{bmatrix}$$

where $a=[a_1,a_2,a_3]^T$. The *hat* map is defined as $\hat{\cdot}:\mathbb{R}^3\to\mathfrak{so}(3)$ such that for every vector $\eta\in\mathbb{R}^3$

$$\hat{\eta}:=\begin{bmatrix} 0 & -\eta_3 & \eta_2\\ \eta_3 & 0 & -\eta_1\\ -\eta_2 & \eta_1 & 0\end{bmatrix},$$

where $\eta=[\eta_1,\eta_2,\eta_3]^T$. The $(\cdot)^\vee$ map is defined as $(\cdot)^\vee:\mathfrak{so}(3)\to\mathbb{R}^3$ and is the inverse of the *hat* map.

Lemma 2.1. *For any $A\in\mathbb{R}^{3\times 3}$ and $b\in\mathbb{R}^3$ the following holds:*

$$\mathrm{tr}(A\hat{b})=(A^T-A)^\vee\cdot b \tag{1}$$

Proof. The proof is presented in Appendix A.

Lemma 2.2. *For any given matrix $Q\in\mathbb{R}^{3\times 3}$, the following relation always holds:*

$$(Q^T-Q)^\vee=\sum_{j=1}^{3} Qe_j\times e_j, \tag{2}$$

where $e_1=[1\ 0\ 0]^T$, $e_2=[0\ 1\ 0]^T$ and $e_3=[0\ 0\ 1]^T$.

Proof. The proof is presented in Appendix B.

3. Problem formulation

The system consists of n identical satellites, each of which has three internal rotors mounted on each of the principal axes of the satellite. The orientation of each satellite is controlled through the actuation of these rotors. *The underlying assumption is that there are no external torques or forces and hence the spatial angular momentum of each satellite is conserved.* The objective is to design a control law which ensures that attitude synchronization is achieved for the satellites. It is assumed that only the first satellite (satellite 1) has the knowledge of the desired orientation.

3.1. Dynamics of the satellite

We first present some notation. For a given satellite,

- The moment of inertia in the body frame is $\mathbb{I}_S=\mathrm{diag}(I_{S_1},I_{S_2},I_{S_3})$, where 1, 2 and 3 are the principal axes.
- The moment of inertia of the jth rotor in the body frame of the satellite is $\mathbb{I}^j$ $(=\mathrm{diag}(I^{j_1},I^{j_2},I^{j_3}))$, $j=1,2,3$.
- The locked-inertia is defined as $\mathbb{I}_L(:=\mathbb{I}_S+\sum_{j=1}^{3}\mathbb{I}^j)$.
- The inertia of the rotors about the principal axes is $\mathbb{I}_R(=\mathrm{diag}(I^{1_1},I^{2_2},I^{3_3}))$.

For the ith satellite,

- The angular velocity of the satellite in its body frame is Ω_i.
- The angular velocity of the rotors in the satellite's body frame is Ω_{r_i}.
- The orientation of the satellite with respect to the inertial frame is given by the rotation matrix R_i.
- The spatial angular momentum of the ith satellite is μ_i.

The dynamics of each satellite is given by [14,2]

$$\dot{R}_i=R_i\hat{\Omega}_i,\quad \mathbb{I}_S\dot{\Omega}_i=R_i^T\mu_i\times\Omega_i-u_i,\quad \mathbb{I}_R(\dot{\Omega}_i+\dot{\Omega}_{r_i})=u_i, \tag{3}$$

where $\mathbb{I}_S=(\mathbb{I}_L-\mathbb{I}_R)$, and $u_i\in\mathbb{R}^3$ is the control applied to the rotors. Note that $\dot{\mu}_i=0$. In terms of body angular momentum Π_i (3) can be written as [2]

$$\dot{R}_i=R_i\widehat{\mathbb{I}_S^{-1}\Pi_i},\quad \dot{\Pi}_i=R_i^T\mu_i\times\mathbb{I}_S^{-1}\Pi_i-u_i,\quad \mathbb{I}_R(\mathbb{I}_S^{-1}\dot{\Pi}_i+\dot{\Omega}_{r_i})=u_i, \tag{4}$$

where $\Pi_i=\mathbb{I}_S\Omega_i$.

4. Control objective and potential function

The control objective is attitude synchronization to a desired attitude R_d. To achieve this, a potential function is designed which represents the coupling between the satellites. The potential function depends on the relative orientation between the satellites. The idea of potential function has been used in [3,23,24], to name a few.

4.1. Relative orientation

The error in orientation of satellite 1 with the desired orientation R_d is given by X_1 which is defined as

$$X_1=R_d^TR_1 \tag{5}$$

The variation of X_1 is

$$\delta X_1=\left.\frac{d}{d\epsilon}\right|_{\epsilon=0}(R_d^TR_1\exp(\epsilon\hat{\eta}_1))=R_d^TR_1\hat{\eta}_1=X_1\hat{\eta}_1,$$

where an admissible variation is $R_1\exp(\epsilon\hat{\eta}_1)$, where η_1 is any vector in $\mathbb{R}^3$.

The relative orientation of satellite i with respect to $(i-1)$ is given by X_i which is defined as

$$X_i=R_{i-1}^TR_i \tag{6}$$

The variation of X_i is

$$\begin{aligned}\delta X_i&=\left.\frac{d}{d\epsilon}\right|_{\epsilon=0}(R_{i-1}\exp(\epsilon\hat{\eta}_{i-1}))^TR_i\exp(\epsilon\hat{\eta}_i)\\ &=\left.\frac{d}{d\epsilon}\right|_{\epsilon=0}\exp(-\epsilon\hat{\eta}_{i-1})R_{i-1}^TR_i\exp(\epsilon\hat{\eta}_i)\end{aligned}$$