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Almost global attitude stabilization of a rigid body for both internal and external actuation schemes

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ABSTRACT

One of the recent developments in attitude control is the notion of almost-global asymptotic stabilization (AGAS) using coordinate-free control laws. In this paper, we examine two aspects related to this line of research. The first is the problem of AGAS with internal actuation. Since all of the results concerning AGAS so far focus on external actuation, we address the internal actuation case and show that there exists a class of control laws that can almost globally stabilize the desired equilibria either by external or internal actuation. The second aspect we analyze is the construction of potential functions leading to AGAS. We show that it is possible to construct such potential functions in such a way that the resulting control torque depends only on two vector observations, thus avoiding the need for explicitly computing the attitude matrix for the purpose of feedback. We also show that these potentials are nothing but the commonly used error functions, namely the modified trace functions.

1. Introduction

Attitude stabilization of a rigid body is a classical and important problem and has been widely studied in the context of controlling spacecraft or underwater vehicles. Even though the state space for this problem is a nonlinear manifold, classically this problem has been analyzed by making use of local representations such as Euler angles or global but redundant representations like quaternions, resulting in differential equations on a Euclidean space. It is only recently that the problem has been analyzed in a 'coordinate-free' framework [17,6,21,7,18,8] where one deals with the dynamics directly on the nonlinear state space, namely the tangent or the cotangent bundle of $SO(3)$. The motivation for doing this, rather than use a coordinate dependent approach, has been documented well in the references cited above.

One of the main advantages of the coordinate-free approach is that it allows a global analysis of the designed feedback control law. However, since the rigid body dynamics evolves on a fiber bundle over a compact manifold, global asymptotic stabilization is not achievable through continuous feedback [5]. The notion of *almost global asymptotic stability* or AGAS (to be defined in Section 2) is the best one can hope to achieve in this situation. Control laws that can achieve AGAS have been indeed developed for the case of external actuation [17,21,7,18,8].

In the case of internal actuation, to the best of our knowledge, the existing literature on rigid body stabilization is solely based upon the coordinate dependent approach. In the classical spacecraft control textbooks such as [30,16,28], the attitude stabilization and the attitude acquisition problems are addressed as a sequence of single axis maneuvers and the system can be treated as linear in this situation. Schaub and Junkins [26] have recently developed a stabilizing control law based on the modified Rodriguez parameterization.

If we assume that the external torques are negligible, the total angular momentum of the system is conserved. In the literature so far, this fact is often accounted for by imposing an algebraic constraint on the evolution of state variables. The coordinate-free approach, by taking into account the conservation of angular momentum, can offer remarkable economy and simplicity in terms of the governing equations and the feedback control law. In this work, we shall adopt this approach and address the stabilization problem using a coordinate-free approach when the actuation is by internally mounted rotors. We show that there exist control laws that achieve AGAS with internal actuation and achieve the same with external actuation, with only a sign change. It is worth noting that this result applies only to a class of control laws and to highlight this, we present an example of a control law which stabilizes with internal actuation, the negative of which *does not* stabilize with external actuation.

Potential functions inspired from the gravitational potential of a spinning top have been used earlier for stabilization of angular velocity of rigid bodies or underwater vehicles [19,25]. These potential functions are capable of stabilizing what are known as

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relative equilibria [22]. On the other hand, trace and the modified trace functions have been used as error functions for total stabilization of rigid body attitude and angular velocity [17,6]. In this paper, we show that these two approaches are closely related. A preliminary version of these results appears in [3].

Control laws have been recently proposed for attitude stabilization using vector observations, without the need for explicitly computing the attitude matrix [24,29,1]. This has an advantage in spacecraft applications that the system can function even if the inertial navigation system fails. In the paper [24], the authors propose a control law for adaptive stabilization using vector observations based on an observer–controller framework. This setup however calls for additional computational effort in an actual application, for the integration of the observer variables. In the paper [29], the authors develop a-priori bounded, velocity-free control laws based on vector observations, using the quaternion representation for the spacecraft orientation. The damping necessary to achieve asymptotic stability is generated by augmenting the rigid body dynamics with an auxiliary system. The resulting closed loop system in [29] might however have a continuum of equilibria, depending on the choice of the observed vectors, as we shall demonstrate. We show, taking motivation from classical rigid body systems such as a spinning top, a method of deriving a coordinate-free, almost globally stabilizing control torque using only two vector observations, without the need for an observer or an auxiliary system. Also, in any globally defined control law, determining the equilibrium points of the closed loop dynamics and the nature of stability at these points is of utmost importance. We show how one can utilize the existing framework of modified trace functions for this purpose and show that the closed system with the control law that we derive has only four isolated equilibria. We also delineate a method of estimating the angular velocity using body-rates of two vector observations, which is in turn used for the damping torque in our control law.

We first review the strategy of AGAS for external actuation in Section 3 using potential or error functions. In Section 4, we then show how the same potential can be used to design stabilizing internal torques. In Section 5 we show how to use vector observations to build modified trace functions. We derive an expression in closed form for the body angular velocity from body-rates of two vector observations in Section 6. We present simulation results in Section 7.

2. Preliminaries

The set of orientations of a rigid body, which is the set of 3×3 proper orthogonal matrices is the Lie group $SO(3)$ [23]. The tangent space at the group identity $e\in SO(3)$, called the Lie algebra of $SO(3)$, is the space of 3×3 skew-symmetric matrices, which is denoted by $\mathfrak{so}(3)$. For any $R\in SO(3)$, we can define a function L_R over $SO(3)$ as $L_R(Q)=RQ$ for every $Q\in SO(3)$. This function, $L_R: SO(3)\longrightarrow SO(3)$ has an inverse and both L_R and L_R^{-1} are differentiable. Hence, L_R is a diffeomorphism and is called 'the left-translation by R'. Every differentiable map has a derivative map or tangent map and T_eL_R, the tangent map of L_R evaluated at e, maps from $\mathfrak{so}(3)$ to $T_RSO(3)$, the tangent space of $SO(3)$ at R. The map $T_eL_R^*: T_R^*SO(3)\longrightarrow\mathfrak{so}(3)^*$, where $T_R^*SO(3)$ is the cotangent space at R and $\mathfrak{so}(3)^*$ is the dual of the Lie algebra, is the dual of T_eL_R.

The vector space $\mathfrak{so}(3)$ can be identified with $\mathbb{R}^3$ using the map $\mathcal{S}:\mathbb{R}^3\longrightarrow\mathfrak{so}(3)$ (also called the *hat* map). For every $a=(a_1,a_2,a_3)\in\mathbb{R}^3$,

$$\mathcal{S}(a)=\begin{bmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{bmatrix}.$$

We can use this map to identify $\mathfrak{so}(3)^*$ with $\mathbb{R}^3$ as follows (see for example [13]). Define $p:\mathfrak{so}(3)^*\longrightarrow(\mathbb{R}^3)^*$, $p=\mathcal{S}^*$, the adjoint of $\mathcal{S}$. If we identify $(\mathbb{R}^3)^*$ with $\mathbb{R}^3$ using the inner product, then $p:\mathfrak{so}(3)^*\longrightarrow\mathbb{R}^3$ is given by

$$p(\rho)\cdot\eta:=\langle\rho,\mathcal{S}(\eta)\rangle, \tag{1}$$

for any $\rho\in\mathfrak{so}(3)^*$. Here the $(\cdot)$ on the left hand side denotes the standard inner product on $\mathbb{R}^3$ and the angles-bracket on the right hand side denotes the dual action of $\mathfrak{so}(3)^*$ on $\mathfrak{so}(3)$. We often denote $\hat{\eta}=\mathcal{S}(\eta)$, for $\eta\in\mathbb{R}^3$, to avoid notational complexity.

We now give a definition of AGAS relevant in our context. This is essentially the notion of almost global stability used in the literature.

Definition 2.1. An equilibrium point $x\in M$ of a vector field X on M is almost globally asymptotically stable if

- x is locally asymptotically stable,
- X has finite number of equilibrium points and the stable manifold of every equilibrium point other than x is a lower dimensional submanifold of M,
- all points in $M\backslash\mathcal{U}$, where $\mathcal{U}$ is the union of stable manifolds of the equilibrium points other than x, converge to x.

3. Review of stabilization by external actuation

The equations of motion of a rigid body with external torque u are given by

$$\dot{R}=R\mathcal{S}(I^{-1}\Pi), \tag{2}$$

$$\dot{\Pi}=\Pi\times I^{-1}\Pi+u, \tag{3}$$

where Π is the angular momentum expressed in the body coordinates and I is the moment of inertia matrix. In a coordinate-free analysis, one directly makes use of the rotation matrices, instead of representing them using coordinates such as Euler angles or quaternions, which implies that one directly considers the dynamics on the nonlinear manifold. For example, consider a top spinning with its point of contact on the ground at rest. The torque due to gravity on the top can be written as a function of the rotation matrix R as $u(R)=-mg\chi\times R^Te_3$, where m and g are the mass and the acceleration due to gravity respectively, χ is the vector from point of contact to the center of mass expressed in the body frame and e_3 is the unit vector in the direction of gravity. Substituting this expression for u in Eq. (3), we get a dynamical system whose state variable consists of R, a 3×3 proper orthogonal matrix and Π, a vector in $\mathbb{R}^3$.

It might seem uneconomical at the outset to directly use proper orthogonal matrices which have nine components, instead of using coordinates such as Euler angles or quaternions which have three or four components. However, the methods of analysis in the coordinate-free set-up seldom require one to express R in terms of its components. The analysis often involves geometric ideas that capture the dynamics by using R as only a symbol to represent the orientation, as we shall demonstrate in this section. We also refer the reader to [8] for a tutorial on the coordinate-free approach.

As it will become clear through the course of this paper, it is often beneficial to look for a feedback control law which can be derived from a scalar valued smooth function $V: SO(3)\longrightarrow\mathbb{R}$. The paper by Koditschek [17] is among the first one to analyze coordinate-free, globally defined control laws, which are derived from what he calls as a *navigation function*. A navigation function is a Morse function on $SO(3)$ and the stabilization properties of the resulting control law were studied using the Riemannian geometry structure that can be given to $SO(3)$. Bullo and Lewis also derive