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A combined support vector machine-wavelet transform model for prediction of sediment transport in sewer

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ABSTRACT

Technical design of sewer systems requires highly accurate prediction of sediment transport. In this study, the capability of the combined support vector machine-wavelet transform (SVM-Wavelet) model for the prediction of the densimetric Froude number (*Fr*) was compared to the single SVM and different existing sediment transport equations at the limit of deposition. The performance evaluation was performed using the R-square (R^2), three relative indexes (*MRE*, *MARE*, *MSRE*) and three absolute indexes (*ME*, *MAE*, *RMSE*). The factors affecting the *Fr* were initially determined. After categorizing them into different dimensionless groups, six different models were found to predict the *Fr*. Comparisons between the obtained results showed that both the SVM and SVM-Wavelet can predict the *Fr* with high accuracy. However, it was found that the SVM-Wavelet (R^2=0.995, MRE=0.002, $MARE$=0.021, $MSRE$=0.001, ME=0.007, MAE=0.086 and $RMSE$=0.114) offers higher performance than the SVM and the existing equations.

1. Introduction

The flow towards the sewer system is commonly accompanied by solid matters. In cases where the flow velocity is less than the minimum required velocity to transport suspended sediment in a fixed channel gradient, the sediments are deposited into the channel bed. Permanent deposition occurs if the flow needed to wash away the deposited sediments does not enter the channel within a specific time period. This will subsequently cause many problems for the sewer. Deposition of solid matters increases the roughness of the channel bed, while decreases the channel cross section. Therefore, there is a need for methods that predict the minimum velocity to prevent sedimentation.

To achieve this, one of the oldest and simplest methods is utilizing the minimum shear stress or velocity. Different shear stress and velocity values have been introduced by Vongvisessomjai et al. [1] for several areas and flow conditions. Considering the fact that these fixed values do not consider the channel and sediment specifications, they do not yield similar results under different flow conditions. These constant values can lead to application of the *Fr* more (or less) than the actual values in the design propose [2,3]. As a result, many methods have been suggested for the design of sewer systems [1,4–7].

Nalluri and Ota [8] presented a model for transporting the sediment at the limit of deposition. Sediment transport equations at the limit of deposition do not yield economical results in large-diameter channels. Thus, Ota and Nalluri [9] used a similar method to develop a model for the sediment transport in large channels. There are a large number of equations to analyze the sediment transfer obtained from different conducted experiments. Banasiak [6] evaluated the prediction capability of each equation through several experiments. In Kuwait, Almedeij and Almohsen [10] provided a number of remarks on the Camp criterion in order to come up with a more flexible equation in terms of the required minimum velocity. Almedeij [11] proposed a self-cleaning design process for the rectangular channels by maintaining the lower limit of the shield stress. Ebtehaj et al. [2] used three different sets of data, including a larger range of densimetric Froude number (*Fr*) and modified the equations put forth by Vongvisessomjai et al. [1] to calculate sediment transport at the limit of deposition.

In recent years, the soft computing approaches have been widely utilized in the sediment transport. Ebtehaj and Bonakdari [12] examined the sediment transport in sewers through the utilization of the artificial neural network (ANN). They developed a

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model to predict the *Fr* and compared the attained results to those of the existing sediment transport equations. It was observed that the ANN model provides higher accuracy than the existing equations. Ebtehaj and Bonakdari [13] used the adaptive neuro fuzzy inference system (ANFIS) to predict the *Fr*. Through the back propagation and hybrid train algorithms, they utilized the sub-clustering and grid partitioning modes to generate the fuzzy inference system (FIS). Ebtehaj and Bonakdari [14] assessed the performance of the particle swarm optimization (PSO) and imperialist competitive algorithm (ICA); two evolutionary algorithms for optimizing the neural networks' weights. Their results indicated that both algorithms were fairly accurate in minimizing the ANN target function.

Recently, the support vector machine (SVM) and wavelet transform (Wavelet) algorithm have been widely used in the civil engineering for the suspended sediment prediction [15], river flow forecast [16], ground water level prediction [17], hydrological prediction [18] and fault classification [19]. In this research work, for the first time the SVM and Wavelet algorithm were coupled to develop a hybrid model for studying the sediment transport in the sewers. The main goal was to increase the accuracy and reliability of the predictions by taking advantage of the specific nature of each approach. In fact, the wavelet transform algorithm was applied to decompose the used data into its various components. Afterwards the attained decomposed components were utilized as inputs for the SVM model. The parameters affecting the *Fr* prediction were initially identified by examining the influential factors in the sediment transport. Thereafter, six different models were proposed to assess the effect of different parameters on predicting the *Fr*. The predictions of the developed SVM-Wavelet model were compared against the SVM model as well as the existing sediment transport equations.

2. Materials and methods

2.1. Support vector machine (SVM)

Assuming a set of data points is given by $\{x_i, d_i\}_i^n$, where x_i is the input space vector of the data sample, d_i is the target value and n is the data size. SVM approximates the function as follows:

$$f(x) = w\phi(x) + b \tag{1}$$

$$R_{SVMs}(C) = C\frac{1}{n}\sum_{i=1}^{n} L(x_i, d_i) + \frac{1}{2}\|w\|^2 \tag{2}$$

where $\varphi(x)$ is a high dimensional feature space that mapped the input space vector x, w is a normal vector, b is a scalar and $C(\frac{1}{n})\sum_{i=1}^{n} L(x_i, d_i)$ represents the empirical error. The parameters w and b can be estimated by minimization of regularized risk function after introduction of positive slack variables ξ_i and ξ_i^* which represent upper and lower excess deviation, respectively.

$$\text{Minimize } R_{SVMs}(w, \xi^*) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}(\xi_i, \xi_i^*) \tag{3}$$

$$\text{Subject to } \begin{cases} d_i - w\varphi(x_i) + b_i \le \epsilon + \xi_i \\ w\varphi(x_i) + b_i - d_i \le \epsilon + \xi_i^* \\ \xi_i^*, \xi_i^* \ge 0, i = 1, \ldots, l \end{cases}$$

where $\frac{1}{2}\|w\|^2$ is the regularization term, C is the error penalty factor used to regulate the difference between the regularization term and empirical error, ε is the loss function which equates to approximation accuracy of the training data point and l is the number of elements in the training data set. Optimality constraints and Lagrange multiplier which can be used to solve Eq. (1) are consequently obtained using a generic function as follow:

Eq. (2) can be solved using the Lagrange multiplier and optimality constraints to obtain a generic function given as:

$$f(x, a_i, a_i^*) = \sum_{i=1}^{n}(a_i - a_i^*)K(x_i, x_j) + b \tag{4}$$

where $K(x_i, x_j) = \phi(x_i)\phi(x_j)$ and the term $K(x_i, x_j)$ is called the kernel function, which is product of the two inner vector x_i and x_j in the feature space $\phi(x_i)$ and $\phi(x_j)$, respectively.

There are four basic kernel functions provided by SVM, namely linear, sigmoid, polynomial and radial basis function. Radial basis function (RBF) has been proved to be the best kernel function due to its computationally efficiency, simplicity, reliability, ease of adaption for optimization and other adaptive techniques as well as its adaptability in handling parameters that are more complex. The non-linear radial basis kernel function is defined as:

$$K(x_i, x_j) = \exp(-\gamma\|x_i \times x_j\|^2) \tag{5}$$

where variable x_i and x_j are vectors in the input space, i.e. vectors of features computed from training or testing data set. Also, γ is the regularization parameter that determines the trade-off between the fitting error minimization and the smoothness of the estimated function. The accuracy of the predictions made by the RBF kernel function depends on the selection of γ, ε and C parameters. The optimal values of these parameters obtaining through the trial and error are 0.45, 0.05 and 0.95, respectively.

2.2. Discrete wavelet transform

The wavelet transform (WT) is a signal processing method obtained using the Fourier series. It represents a mathematical expression for decomposing a time series' frequency signal into different components. One of its advantages over the Fourier transform is the perfect analysis of the resulting decomposed components with well-scaled resolution, which helps in improving the capacity of the study model because it captures the needed information at different levels. It is suitable for analyzing data in the frequency and time domains due to its capability of extracting data from non-periodic and transient signals; thus, it is very useful in time-frequency localization.

Continuous wavelet transform (CWT) of a signal $f(t)$ is a time-scale technique of signal processing that can be defined as the integral of all signals over the entire period multiplied by the scaled, shifted versions of the wavelet function $\psi(t)$, given mathematically as:

$$W_x(a, b, \psi) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt \tag{6}$$

where $\psi(t)$ is the mother wavelet function, a is the scale index parameter (i.e., inverse of the frequency), and b is the time shifting parameter, also known as translation. The discrete wavelet transform (DWT) can be derived by discretizing Eq. (6), where the parameters a and b are obtained by $a = a_0^m$, $b = na_0^m b_0$.

The variables n and m are integers. Replacing a and b in Eq. (6) gives:

$$W_x(a, b, \psi) = a_0^{-m/2}\int_{-\infty}^{+\infty} f(t)\psi^*(a_0^{-m}t - nb_0)dt \tag{7}$$

In this study, the wavelet analysis was conducted to decompose the used data into various components, after which the decomposed components were used as inputs for the SVM model.

The SVM-Wavelet models were developed by integrating the