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Direct solutions for normal depth in parabolic and rectangular open channels using asymptotic matching technique

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ABSTRACT

Normal flow depth is an important parameter in design of open channels and analysis of gradually varied flow. In open channels with parabolic and rectangular cross-sections, the governing equations are nonlinear in terms of the normal depth and thus solution of the implicit equations involves numerical methods. In current research explicit solutions for these channels have been obtained using asymptote matching technique. For the parabolic channel, the maximum error of proposed equation for normal depth is less than 0.07% (near exact solution). But, in rectangular channels, the maximum error of proposed equation for normal depth is less than 1.94% which is not very accurate. The efficiency of the asymptote matching technique can be considerably improved by adding a power-law function between two asymptotes. For rectangular channel a new solution for normal flow depth is developed using the improved asymptote matching technique proposed in this research. The maximum error of this full range solution is less than 0.12%. The results showed that the improvement in proposed solution is substantial. Proposed full range solutions have definite physical concept, high accuracy and easy calculation and are well-suited for manual calculations and computer programming.

1. Introduction

Normal flow depth plays an important role in the design of open channels. Determination of the normal flow depth is an important task for efficient hydraulic design and hence many researches have been carried out in this subject. The simplest open channel flow cross section is a rectangle. Rectangular channels are widely used for open channel flow. Open channels with parabolic cross-sections are often a quite good approximation of the real geometry of natural rivers. Technology is also available for constructing this shape of channels. The parabolic open channel is widely used in irrigation area, especially in the farmland irrigation projects of cold regions [1]. There are no analytical solutions to explicitly compute the normal water depths in rectangular and parabolic open channels. Thus any effort for presenting direct normal depth calculation with high accuracy would be of practical importance. Normal flow depths in rectangular and parabolic open channels are presently obtained by numerical method, manual trial-and-error method (time-consuming), graphical method (low accuracy owing to its log-scale representation) or by using explicit regression-based equations.

Babaeyan [2] presented graphical solutions for the normal depths of round-corner rectangular and parabolic channel sections; however, his solution is not suitable for practical applications. Various explicit equations for determining the normal depth have been developed for rectangular cross-section with various degrees of estimation errors and different application range (among them [3–7]). Swamee and Rathie [3] proposed two converging infinite series solutions (based on Lagrange's inversion theorem) that help in evaluating the normal depth in wide and narrow rectangular cross sections. When two different solutions are given for a cross section, a limit of applicability should be determined. Based on the truncation of an iterative algorithm, Srivastava [4] showed that a fitted series would be more accurate than a truncated one. Some researches have also been down on normal depth calculation of parabolic channels. For parabolic cross section, Achour and Khattaoui [8] derived three explicit solutions for the normal depth, depending on the value of the relative normal depth. The proposed solution is not a unified one and its relative error reaches up to 1%. Li and Gao [1] proposed an explicit two-step solution for estimating the normal depth of parabolic channel with a relative error less than 0.34% for the ratio of width to depth between 0.2 and 20. It would be more useful to have a simple and accurate solution rather than one which is more complicated (two-step solution).

The main focus of this research is on accurate and explicit full range solutions for normal flow depth. In current research, using the asymptote matching technique, accurate and direct solutions have been developed to determine the normal depths of

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Notations

A	Cross-section area of flow (m^2)
B	Bottom width of rectangular channel (m)
g	Gravitational acceleration (m/s^2)
k	Parabolic parameter (m^{-1})
n	Manning roughness coefficient ($m^{-1/3}$ s)
p	Matching coefficient
P	Wetted perimeter of flow (m)
Q	Channel flow discharge (m^3/s)
R	Hydraulic radius (m)
S_0	Channel longitudinal bed slope (m/m)
X	Abscissa (m)
y	Channel flow depth (m)
Y	Ordinate (m)
y_n	Normal flow depth (m)
α	Fitting coefficient
β	Fitting coefficient
ν	Kinematic viscosity (m/s^2)
η	Non-dimensional flow depth in parabolic channel [$=4ky$]
ζ	Non-dimensional flow depth in rectangular channel [$=y/B$]
λ	Unit conversion constant
ε_p	Non-dimensional discharge for normal flow depth computations in parabolic channel
η_n	Non-dimensional normal flow depth in parabolic channel [$=4ky_n$]
ζ_n	Non-dimensional normal flow depth in rectangular channel [$=y_n/B$]
ε_r	Non-dimensional discharge for normal flow depth computations in rectangular channel
ε	Channel surface roughness (m)
η_{n0}	Lower asymptote for parabolic section (non-dimensional)
$\eta_{n\infty}$	Upper asymptote for parabolic section (non-dimensional)
ζ_{n0}	Lower asymptote for rectangular section (non-dimensional)
$\zeta_{n\infty}$	Upper asymptote for rectangular section (non-dimensional)
$\zeta_{n\beta}$	Power-law function for rectangular section [$=\alpha\varepsilon_r^{\beta}$ non-dimensional]

Subscript

"n"	Denotes the uniform flow condition

rectangular and parabolic cross sections. Proposed solutions cover entire practical range. The proposed solutions are preferable to previous presented solutions in terms of accuracy, generality and simplicity.

2. Geometric properties

A parabolic channel (Fig. 1a) is described by below equation:

$$Y = kX^2 \tag{1}$$

where Y=ordinate (m); X=abscissa (m); and k=a parameter for which the function takes different shapes. Note that, the unit of k is m^{-1}. The flow area of the channel, A, for the flow depth, y, can be computed as

$$A = 2\int_0^{\sqrt{\frac{y}{k}}} (y - kX^2)dX = \frac{4}{3\sqrt{k}}y^{3/2} = \frac{\eta^{3/2}}{6k^2} \tag{2}$$

where $\eta=4ky$.

The wetted perimeter, P, can also be obtained as follows:

$$P = 2\int_0^{\sqrt{\frac{y}{k}}} \sqrt{1+4k^2X^2}\,dX = \frac{\sqrt{\eta(1+\eta)} + \ln(\sqrt{\eta} + \sqrt{1+\eta})}{2k} \tag{3}$$

The flow area, A, of a rectangular channel for the flow depth, y, is equal to By in which B is the bottom width of the channel (Fig. 1b). The wetted perimeter, P, is also equal to $B+2y$.

3. Governing equations and their non-dimensional forms

In gradually varied flow computations, also design and operation of open channels it is required to determine normal/uniform flow depth, y_n. The normal flow depth is occurred when the increase in energy due to elevation drop is balanced by friction losses along the open channel.

The Manning equation for uniform flow in an open channel with hydraulically rough surfaces is given by [9–11]

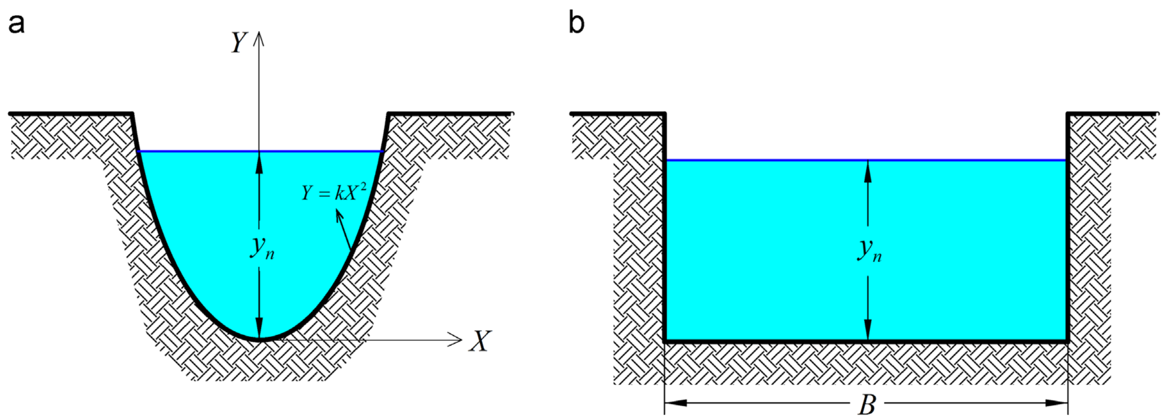

Fig. 1. Cross-section geometry for (a) parabolic open channel and (b) rectangular open channel.