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Optimal Regulation for Dynamic Hybrid Systems Based on Dynamic Programming in the Case of an Intelligent Vehicle Drive Assistant

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Abstract: This contribution proposes an application of an optimisation algorithm to be implemented in an intelligent drive assistant. A vehicle dynamic model is introduced and after the characterisation of its considered nonlinearities some elements of the background concerning "Dynamic Programming" are given together with the proposed algorithm. A two-point boundary value optimization problem in term of velocity is proposed with its solution using the well-known "Dynamic Programming" method. The novelties of this approach consists of a technique which leads to a global optimum. In fact, this technique searches throughout the whole state-space for it. Because of the nonlinearity of the considered problem, using numerical methods seems to be the only way solving it. Using "Dynamic programming" such kind of a complex problem is solved by dividing it into a collection of simpler subproblems. At the end, an optimal feedback controller for the angular position of the throttle valve of the engine is proposed. Simulation results are discussed together with possible application aspects such as "Curse of Dimensionality" and an explicit analysis concerning the calculation effort.

Keywords: Optimal control; Autonomous vehicles; Hybrid models; Modelling;

1. INTRODUCTION AND MOTIVATIONS

Dynamic systems are usually classified as continuous or discrete dynamic ones. However, real systems can often not be clearly classified into one of these categories. Most of real applications dynamic systems contain continuous and discrete dynamics as well. This mixture of continuous and discrete dynamics is called a hybrid dynamic system (or short: hybrid system). Therefore, hybrid dynamic systems exhibit continuous and instantaneous changes, having features of continuous-time and discrete-time dynamical systems. Many examples are given also from physics and even more they can be found in the modern technologies in which continuous variables are always mixed with no continuous ones. Very often just the necessity of reconfigurations of the controller as well as the observer generates many difficulties as shown in Mercorelli (2012). A prime example of a dynamic hybrid system is represented by the longitudinal dynamics of a vehicle with stepped transmission. Here, the longitudinal dynamics is continuous and the selected gear is a discrete variable. The consideration of a system as a hybrid one allows optimal arrangements, incorporating all existing dynamics to be designed, Goebel et al. (2009) and Goebel et al. (2012). Here, "Optimal" has always to be seen in terms of a cost function. For example, energy-optimal or time-optimal controllers are conceivable. In the design of optimal schemes, there are two basic approaches: the first one is the "Pontryagin's Maximum Principle" and the second one is the "Dynamic programming". The "Pontryagin's Maximum Principle" provides conditions for an optimum. However, these conditions are often only necessary ones. Therefore, these conditions generally lead only to local optima and often just only continuous solutions. Another problem is that "Pontryagin's Maximum Principle" leads only to optimal controls and not to optimal feedback laws as "Dynamic Programming" does. Pontryagin and Gamkrelidze (1986), Kirk (1970) and Bertsekas (1995). The other way to solve optimal control problems, in addition to the "Maximum Principle", is represented by the "Dynamic Programming" which was transferred into the field of control theory from physics by Richard Bellman. In this case, the problem is solved dividing it into subintervals Bellman and Kalaba (1965). The rule here is: "If an optimal trajectory is broken into two pieces, then the last piece is itself optimal". This principle being seemingly trivial has in the use a significant impact on the number of necessary calculations, because it reuses already known results which have been already calculated. In "Dynamic Programming" all admissible trajectories are calculated and this leads to global optima. In this paper an optimal problem is presented and solved for hybrid systems based on "Dynamic Programming". For this purpose, as an application, the longitudinal dynamics of a vehicle with stepped transmission is presented together with the design of an optimal controller for the throttle valve. Intelligent vehicles are widely researched fields in which energy in term of research is involved. This is due to the development of the electrical vehicles which need intelligent drive assistants, Qu. et al. (2012) and Mercorelli et al. (2014). The contribution in Mercorelli (2014b) is devoted to calculate the optimal velocity profile to be given as a reference signal to the controller and it also combines the results presented in Mercorelli (2014a). In Mercorelli (2014b) the considered optimization problem is formulated as a two-point boundary value optimization problem that clearly takes the structural physical constraints and the properties of the system into account in which the proposed optimization technique is similar to that presented in Fabbrini

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et al. (2012). In Fabbrini et al. (2012) a direct method to optimise trajectories of an actuator is proposed and tested. In this paper a two-point boundary value optimization problem in term of velocity is proposed with its solution using the well known "Dynamic Programming" method. Because of the nonlinearity of the considered problem "Dynamic programming" is a good approach to solve such kind of a complex problem by breaking it down into a collection of simpler subproblems. The existing nonlinearities between angular position of the throttle valve, the engine cycles in term of rounds per minute (rpm) and the motor torque are expressed with the help of the characteristic lines of the motor. So, the optimal angular position profile is also calculated. In the last years a clear interest is shown for the control of the throttle valve, see Mercorelli (2009), Montanaro et al. (2014), Kadir et al. (2014) and Vargas et al. (2014). Therefore, an optimal controller for the angular position of the throttle valve is proposed. The paper is organised in the following way. Section 2, after a short overview on the necessary background, proposes the structure of the optimisation procedure. The next Section 3 considers the mathematical model of the vehicle. In Section 4 an optimal controller for the throttle valve is analysed. Section 5 is devoted to the analysis of the simulated results. Conclusions and outlook close the paper.

The main nomenclature

$\mathbf{x}(N-k)$: discrete state variable vector
x_1: position
x_2: velocity
$u(N-k)$: discrete input signal
T: final time
m: mass of the vehicle
J: cost function
F_f: force induced by the rolling friction
F_a: aerodynamic drag force
F_d: the downhill force
$F_m(u_1)$: force produced by the cars powertrain
n_{mot}: engine cycles
g: gravitational force
f_r: air resistance
cw: drag coefficient
α: local road inclination
A: frontal surface of the vehicle
i_A: ratio of axle drive
$i_G(u_2)$: gear ration
η: powertrain efficiency
ρ: roll resistance
$M_{mot}(u_1, n_{mot})$: motor torque as function of throttle valve angular position and engine cycles

2. BACKGROUND AND PROPOSED ALGORITHM

Since the proposed controller is based on the ideas of hybrid dynamic systems and "Dynamic Programming", at this point the fundamentals of these approaches are presented.

2.1 Hybrid Dynamic Systems

The basis for this subsection can be found in Goebel et al. (2009) and Goebel et al. (2012). A Hybrid Dynamic System (HDS) is a combination of continuous and discrete system behaviour. Following Goebel et al. (2009) and Goebel et al. (2012) such systems can be modeled by considering typical system descriptions of continuous and discrete systems. A continuous system can be described by $\dot{x} = f(x), with\ x \in \Re^n$. Using the constraints that $x \in C, with\ C \subseteq \Re^n$ and a set valued mapping of the right side, the system becomes $\dot{x} \in F(x), with\ x \in C$. A similar procedure can be done for a discrete system $x^+ = g(x), with\ x \in \Re^n$. With the constraints $x \in D, with\ D \subseteq \Re^n$ and a set valued mapping of the right side the system becomes $x^+ \in G(x), with\ x \in D$. By combining these two systems, one gets a general description for a hybrid dynamic system.

$$\dot{x} \in F(x), with\ x \in C \tag{1}$$

$$x^+ \in G(x), with\ x \in D \tag{2}$$

2.2 Dynamic Programming

"Dynamic Programming" is a computational procedure to reduce the number of calculations needed for e.g. calculation of an optimal control law (as it will be the case here). The basis of "Dynamic Programming" is formed by Bellman's "Principle of Optimality" which says: 'An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.'Bellman and Kalaba (1965) This seems to be trivial but in practice it dramatically reduces the calculations needed. For more detail on the reduction of calculations, see Knowles (1981). Because in this paper the aim is to propose a feedback controller, another principle is needed. This principle is the "Principle of Causality" and it says that the state $x(N-k)$ and the control $u(N-k)$ determine the state $x(N-k+1)$ uniquely. Together these two principles lead to the "Principle of Optimal Feedback Control", which states that: Because the control depends on the state, the result is a feedback control law, see Dyer and McReynolds (1970). By using these principles, by discretising the state space and by quantising the states and control, the optimal controller for a hybrid dynamical system can be calculated using "Dynamic Programming" as described in e.g. Knowles (1981). The main problem using "Dynamic Programming" is the "Curse of Dimensionality" which in case of complex system can lead to overflow of random access memory of the calculating computer. In the application there are no problems using this approach, because the controller needs little memory and can be integrated using a table lookup.

2.3 General problem setup and calculation procedure

The general problem is to find an optimal control, which depends on the state, such that a cost function is minimal and all constraints hold. Where the constraints are the hybrid dynamic system and some other physical constraints like for example a minimum and a maximum for the engine cycles. The mathematical problem can be formulated as:

$$J(x(t), u(t), T) = \upsilon(x(T)) + \int_0^T \phi(x(t), u(t))dt \tag{3}$$

subject to:

$$\dot{x} \in F(x), with\ x \in C \tag{4}$$

$$x^+ \in G(x), with\ x \in D \tag{5}$$